SM221, 4042, Examination 1

Spring Semester 2007

Conditions: You may use one page of notes for reference, and a calculator line the Voyage200 for computations. You may consult only with the instructor about questions on the examination.

Answer the question s on the paper provided, one question per page. Return the examination with your solutions to the problems.

Credit will be awarded for the solution of a problem—the lines of development leading to the answer-- not merely an answer to a problem. In cases of miscalculation, partial credit may be awarded based upon the extent of the development of an answer to a problem.

Notations: We represent points in 3-space in Cartesian (rectangular) coordinates by the ordered triple $P(x,y,z)$, in cylindrical coordinates by $P(r,\theta,z)$ and in spherical coordinates by $P(\rho,\theta,\phi)$.

1. While taking R&R on the Apollo 14 mission, then Capt. Alan Shepard tried a round of golf. Because of the Moon's smaller size and "mass concentrations", while in flight, with the origin of the coordinate system at the tee, the horizontal (x) axis under the line of flight of the ball, the vertical (y) axis up, and with the ***i*** and ***j*** unit vectors specified accordingly (see picture on board), assume that the acceleration that the ball experienced while in flight was

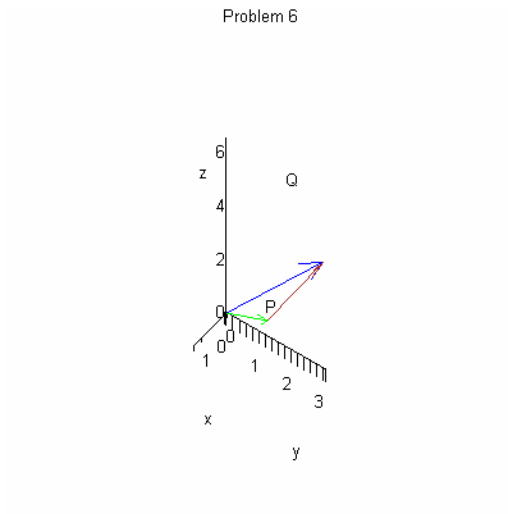
$$\mathbf{a}(t) = -\left(6+\cos(t)\right)\mathbf{j}\ \text{ ft/s}^2$$

He chipped the ball with a launch speed of about 100 ft/s, with a loft angle of about 30°. Taking the tee as the origin,

a) *(12 points)* What are the velocity and position vector functions governing the flight of the golf ball?
b) *(4 points)* On TV, the time of flight for the ball was clocked at about 16.6346 seconds. Does your model for the motion support this value, or was it as optical illusion? Justify your assertion, using your model.
c) *(4 points)* If the time of flight for the ball was 16.6346 seconds, how far downrange did the ball travel?
d) *(4 points)* What was its speed at impact?
e) *(6 points)* How far through the air did the ball travel along its trajectory?

Ed note: At the time, people were wow'ed by the video of the shot (still available on the web). Ironically, we've grown so jaded to the wonders and whimsy of the universe that hardly anyone took notice when a Russian cosmonaut launched a golf ball into orbit from the International Space Station in 2006.

2. *(12 points)* From the origin $O(0,0,0)$ and the two points $P(1,2,2)$ and $Q(0,3,4)$, we can construct three vectors $\mathbf{a} = \overrightarrow{OP}, \mathbf{b} = \overrightarrow{OQ},$ and $\mathbf{c} = \overrightarrow{PQ}$, as shown in the figure:



a) Compute the angle $\angle POQ$ between the vectors $\mathbf{a}$ and $\mathbf{b}$.

b) Compute the scalar projection of vector $\mathbf{b}$ onto vector $\mathbf{a}$.

c) Compute the analytical specification of a vector perpendicular to the vectors $\mathbf{a}$ and $\mathbf{b}$.

3. *(8 points)* Does the line

$$x(t) = t - 3$$
$$y(t) = t - 6$$
$$z(t) = t + 3$$

Intersect the plane

$$x - 2y + z = 9\ ?$$

If so, specify the coordinates of the point of intersection; if not, briefly justify why not?

4. *(10 points)* Construct the equation for a plane that *intersects* the line of problem #3 and is perpendicular to it.

5. *(12 points)* Describe and sketch the following geometric objects:

a) $x^2 + y^2 - z + 1 = 0$,

b) $y^2 + z = 5$,

c) the intersection of a) and b).

6. *(12 points)* Does the motion

$$\mathbf{r}(t) = 2\cos(t)\,\mathbf{i} + \sqrt{2}\sin(t)\,\mathbf{j} + \left(4 + \cos^2(t) - \sin^2(t)\right)\mathbf{k}$$

a) traverse the curve described in problem #5c)? If so, demonstrate why; if not, demonstrate why not.

b) What is its velocity vector function?

c) What is its acceleration?

7. *(16 points)* Sketch (including a coordinate system) or describe in words the surfaces represented by the following equations. If describing the surface in words, be specific about the location, orientation and shape of the surface. ***Hint***: you may represent the equation in any coordinate system that is most comfortable for you.

a) $r^2 + z^2 = 4$
b) $r^2 - z^2 = 4$
c) $\rho \cos(\phi) = 4$
d) $r \cos(\theta) = 4$