SM221 4042
Spring Semester 2007
Examination 2

Conditions: You may use symbolic calculators, such as the TI-92 or Vantage 200 series, to assist you in your computations.

1. *(15 points)* Compute $\frac{\partial g}{\partial u}(u,v)$, if $g(u,v) = f(x(u,v), y(u,v))$, and

$$f(x,y) = x\sin(2y)$$
$$x(u,v) = \ln(u+v)$$
$$y(u,v) = e^{uv}$$

2. *(20 points)*

a) Suppose $f(x,y) = x\cos(y^2)$, $x = x(t)$, $y = y(t)$, and $g(t) = f(x(t), y(t))$.

Find $\frac{dg}{dt}(1) = \frac{dg}{dt}(t)_{|t=1}$, if

$x(0) = 0$	$x(1) = 0.524$	$x(2) = 0.866$
$y(0) = 0$	$y(1) = 0.8863$	$y(2) = 1.047$
$\frac{dx}{dt}(0) = 1.047$	$\frac{dx}{dt}(1) = 0.866$	$\frac{dx}{dt}(2) = -0.524$
$\frac{dy}{dt}(0) = 0.524$	$\frac{dy}{dt}(1) = 0.524$	$\frac{dy}{dt}(2) = 0.524$

b) Forge the formal chain rule with which you'd compute $\frac{\partial L}{\partial u}(u,v)$, if the function $L = f(x,y,z)$, where $x = x(t),\ y = y(t),\ z = z(t)$, and where $t = t(u,v)$. **DO NOT** attempt to execute the chain rule; rather, just set it up.

3. *(30 points)* Given the function

$$F(x,y) = \sqrt{x^2 - y^2}$$

a) Build the algebraic equation for and sketch the level set containing the point $P(5,4)$.

b) Evaluate the gradient of $F(x,y)$ at the point $P(5,4)$ and position the vector at the field point on the sketch of the level set of part a.

c) If you move from the field point towards the point $Q(6,6)$, what is the derivative of the function in that direction?

d) In what direction (**unit vector**) must you move in order for the function to increase in value as rapidly as possible? What is that maximum rate of increase?

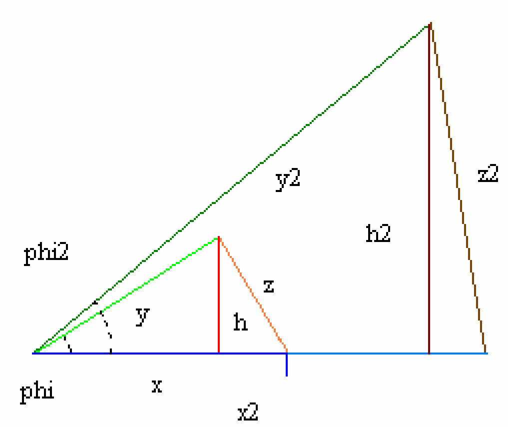
e) What is the equation of the line tangent to the level curve for $F(x, y)$ at the point $P(5,4)$?

4. *(15 points)* The following table samples the value of a function $f(x, y)$ in the vicinity of the point $(x, y) = (2,1)$:

		y		
		0.9	1.0	1.1
	1.9	4.34	4.61	4.94
x	2.0	4.73	5.00	5.33
	2.1	5.14	5.41	5.74

Estimate the directional derivative of $f(x, y)$ at the point $(2,1)$ in the direction of $(1.9,1.1)$.

5. *(20 points)* The area of any triangle is given by the time honored formula $A = bh/2$, where b is the length of the base of the triangle and h is its altitude or height. The relation between the sides of a general triangle, its altitude and the angle included between two side is shown by the figure



When the base x of the triangle has length 40 in, when the side y above the base has length 50 in, and the contained angle ϕ is $\pi/6$ radians, suppose that the length of the base x is *increasing* as a rate of 3 in/sec, the contained angle ϕ is increasing at a rate of 0.05 radians/sec, and the length y of the side opposite the base is *increasing* at the rate of 2 in/sec.

a) What's the rate of change of the altitude (height) of the triangle?

b) What's the rate of change of the area of the triangle?