SM221 Examination 4
Spring Semester 2007

Conditions. You may use one page of notes and a Voyage200, TI92, or any other symbolic manipulator or device to assist you in your computations. Show your development, as it will be the basis for partial credit. You may not consult or discuss the examination with any person other than the instructor.

1. A wire in the shape of a the curve $C: y = x^3$ extends from the point $(x, y) = (-1, -1)$ to the point $(x, y) = (1, 1)$. It is charged with a "linear charge density" of $f(x, y) = x + y^2$ (coulombs/meter). Set up a parameterized integral that determines the net charge on the wire. ***Do not evaluate*** the integral, just set it up.

2. A proton is given energy by subjecting it to an electric field ***E****(x,y)*. The force exerted by the field on the proton is given by the relation

$$\boldsymbol{F}(x, y) = q\,\boldsymbol{E}(x, y),$$

where q is the charge on the proton. For this problem take $q = 1$ (esu).
Let the electric field be

$$\boldsymbol{E}(x, y) = 2 \cdot 10^4 e^{2y} \left(x\,\boldsymbol{i} + x^2\,\boldsymbol{j}\right) \text{ (V/m)}$$

In a cyclotron, by an ingenious use of magnetic fields the proton can be made to move in this electric field in the *x-y* plane along an Archimedean spiral C, described by the equations

$$y = x \tan\left(\sqrt{x^2 + y^2}\right),$$

or, in polar coordinates,

$$r = \theta,$$

and as shown in the figure 'Question 2'. The energy W gained by the proton in this process is determined by the line integral of the force exerted by the electric field on it as it moves over the curve,

$$W = \int_C \mathbf{F} \cdot \mathbf{d}r \text{ (eV)}$$

Find the energy (eV) the proton gains as it moves from the entry port at the origin to the exit port at $(x, y) = (12\pi, 0)$.

3. You create a reflecting surface as that segment of the upper hemisphere of the sphere $x^2 + y^2 + z^2 = 25$ that is contained in the cone $3z^2 = x^2 + y^2$, as shown in the figure 'Question 3'. The unit of measure is centimeters (cm)

a) What is the area of the surface of the reflector $\left(\text{in } cm^2\right)$?

b) The point $(0,3,4)$ lies on the reflector. What is the equation of the plane tangent to the surface at that point?

4. Evaluate the line integral

$$\int_C y^2 dx + 2x^2 dy$$

Over the curve *C* that runs along the quarter circle $x^2 + y^2 = 4$ clockwise from $(0,2)$ to $(2,0)$, along the *x* axis from $(2,0)$ to the origin, then up the *y* axis from the origin to the original starting point $(0,2)$, which is depicted in the figure 'Question 4'.

5. At 0900, the temperature in CH334 varies with location in the room, and may be modeled by the scalar field (real-valued function)

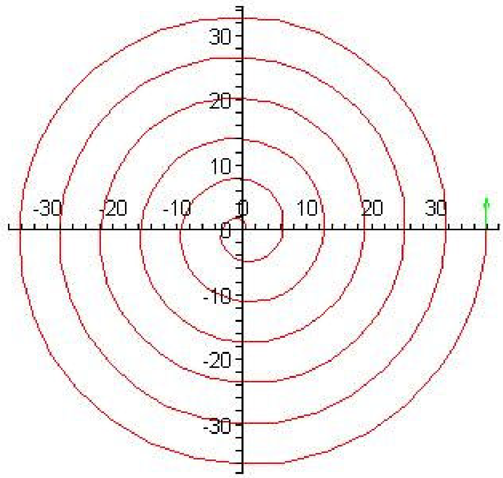
$$T(x,y,z) = x^2 + y^2 - z^2$$

The variation in temperature produces a *heat flux* vector field, defined by

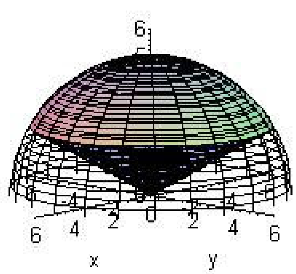
$$\boldsymbol{F}(x,y,z) = -\nabla T(x,y,z)$$

a) Specify the heat flux vector field in terms of its components.
b) Is the heat flux vector field a "curl free" vector field? Justify your assertion by analysis or computation.
c) Is the heat flux vector field a "divergence free" vector field? Justify your assertion by analysis or computation.

Question 2



Question 3



Question 4