Part 1: Multiple Choice (50%). For each question, circle the letter for the best answer.

1. Let \( A, B, C, D, \) and \( E \) be the vertices (in order) of a pentagon with each side of length 1. Then \( \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} \) equals:
   (a) \( \overrightarrow{AD} \)  
   (b) 3  
   (c) \( \overrightarrow{AB} - \overrightarrow{CD} \)  
   (d) \( \overrightarrow{DA} \)  
   (e) \( \overrightarrow{AB} + \overrightarrow{CD} \)

2. If \( |y| = 4, \ |z| = 5, \) and \( y \cdot z = 0, \ |y \times z| \) is:
   (a) 0  
   (b) \( \sqrt{10} \)  
   (c) \( \sqrt{20} \)  
   (d) 10  
   (e) 20

3. The line through the two points \((-1,2,1)\) and \((1,1,2)\) also contains the point:
   (a) \((0,0,0)\)  
   (b) \((0,3,3)\)  
   (c) \((3,0,3)\)  
   (d) \((3,3,0)\)  
   (e) \((3,2,3)\)

4. The angle between the vectors \( \langle 2, -2, 1 \rangle \) and \( \langle 3, 0, 0 \rangle \) is approximately:
   (a) 0.383 rad  
   (b) 0.841 rad  
   (c) 0.931 rad  
   (d) 6 rad  
   (e) 48.2 rad

5. The plot for the equation \( x^2 + 4y^2 + 9z^2 = 36 \) is a:
   (a) sphere  
   (b) cylinder  
   (c) ellipsoid  
   (d) parabolic cylinder  
   (e) plane

6. The equation of the line through the point \((1,3,-1)\) perpendicular to the plane \( 2x - y + z = 3 \) is given by:
   \[ x = 1 + 2t \quad x = 2 + t \quad x = -1 + 2t \quad x = -2t \quad x = 2 - t \]
   (a) \( y = 3 - t \)  
   (b) \( y = -1 + 3t \)  
   (c) \( y = -3 - t \)  
   (d) \( y = t \)  
   (e) \( y = -1 - 3t \)

7. Which of these planes is parallel to the line \( x = 2 - t, \ y = -2 + \frac{1}{2} t, \ z = 1 + 2t \)?
   (a) \( x - \frac{1}{2} y - 2z = 2007 \)  
   (b) \( 2x - 2y + z = 2007 \)  
   (c) \( x - 2y - \frac{1}{2} z = 2007 \)  
   (d) \( -\frac{1}{2} x + \frac{1}{2} y - z = 2007 \)  
   (e) \( 2x + z = 2007 \)

8. Which of these planes is perpendicular to the line \( x = 2 - t, \ y = -2 + \frac{1}{2} t, \ z = 1 + 2t \)?
   (a) \( x - \frac{1}{2} y - 2z = 2007 \)  
   (b) \( 2x - 2y + z = 2007 \)  
   (c) \( x - 2y - \frac{1}{2} z = 2007 \)  
   (d) \( -\frac{1}{2} x + \frac{1}{2} y - z = 2007 \)  
   (e) \( 2x + z = 2007 \)
9. Suppose \( \vec{u} \) and \( \vec{w} \) are unit vectors, and the angle between them is 30°. What is the magnitude of \( |\vec{u} \times \vec{w}| \)?

(a) 0  (b) 1  (c) \( \sqrt{3} \)  (d) \( \frac{1}{2} \)  (e) \( \frac{\sqrt{3}}{2} \)

10. Suppose \( \vec{v} \) and \( \vec{w} \) are vectors. Which of the following expressions is a vector?

(a) \( \vec{v} \cdot \vec{w} \)  (b) \( |\vec{v}| + \vec{w} \)  (c) \( \vec{v} / \vec{w} \)  (d) \( |\vec{v}| \vec{w} \)  (e) \( \vec{v} + \vec{w} \)

11. If \( \vec{v} = \langle 0,2,-1 \rangle \) and \( P = (0,2,-1) \), then \( 0(x-4) + 2(y-1) - (z-2) = 0 \) is the equation of:

(a) a line parallel to \( \vec{v} \)  (b) a line through \( P \)  (c) a plane parallel to \( \vec{v} \)

(d) a plane through \( P \)  (e) a plane perpendicular to \( \vec{v} \)

12. Which of the following is a unit vector?

(a) \( \langle 2,1,-2 \rangle \)  (b) \( \left\langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \right\rangle \)  (c) \( \langle 1,1,1 \rangle \)  (d) \( \langle 3,3,3 \rangle \)  (e) \( \left\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right\rangle \)

13. The vertices of a rectangle are A, B, C, and D (in order). The vector \( \overrightarrow{AB} - \overrightarrow{BC} \) is equal to:

(a) \( \overrightarrow{AC} \)  (b) \( \overrightarrow{DB} \)  (c) \( \overrightarrow{AD} \)  (d) \( \overrightarrow{BC} \)  (e) \( \overrightarrow{CB} \)

14. A vector perpendicular to both \( \langle 1,2,3 \rangle \) and \( \langle 2,1,-1 \rangle \) is:

(a) \( \langle -5,7,-3 \rangle \)  (b) \( \langle -2,1,0 \rangle \)  (c) \( \langle 0,-3,2 \rangle \)  (d) \( \langle 3,3,3 \rangle \)  (e) \( \langle 0,1,0 \rangle \)

15. The point on the Cartesian coordinates \( (x,y,z)=(2,2,0) \) has the spherical coordinates \( (\rho, \theta, \phi) = \)

(a) \( \left( \sqrt{8}, \frac{\pi}{4}, \frac{3\pi}{2} \right) \)  (b) \( \left( \frac{8}{\sqrt{2}}, \frac{\pi}{4}, \frac{\pi}{2} \right) \)  (c) \( \left( \sqrt{8}, \frac{\pi}{2}, \frac{\pi}{4} \right) \)  (d) \( \left( \frac{8}{\sqrt{2}}, \frac{\pi}{4}, \frac{\pi}{2} \right) \)  (e) \( \left( \frac{8}{\sqrt{2}}, \frac{\pi}{2}, \frac{\pi}{2} \right) \)

Part 2: Free Response (50%). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.

16. Given the vectors \( \vec{u} = \langle 4,3,-12 \rangle \) and \( \vec{v} = \langle -2,1,2 \rangle \) find

(a) \( 2\vec{u} - 3\vec{v} \)  (b) \( \vec{u} \cdot 3\vec{v} \)  (c) \( 2\vec{u} \times 3\vec{v} \)

(d) a unit vector in the direction of \( 2\vec{u} - 3\vec{v} \)

(e) \( \text{comp}_v \vec{u} \)
17. (a) Find the parametric equations for the line through (1,-1,0) and (2,2,1).
(b) Find the equation of the plane through (1,2,3), (2,5,4), and (0,4,-1).
(c) Verify that your line and your plane are parallel.
(d) Find the distance between any point on your line and your plane.

18. Consider the points $A=(5,0,0)$, $B=(0,3,0)$, and $C=(0,0,2)$ which are the vertices of a triangle:
   (a) Compute $\overrightarrow{CA} \cdot \overrightarrow{CB}$
   (b) Determine angle $C$ to the nearest degree.
   (c) Find $\text{proj}_{\overrightarrow{CA}} \overrightarrow{CB}$
   (d) Compute $\overrightarrow{CA} \times \overrightarrow{CB}$.
   (e) Find the equation of the plane $E$ that contains $A$, $B$, and $C$.
   (f) Find the line through the origin perpendicular to the plane $E$.
   (g) Find the area of the triangle formed by $A$, $B$, and $C$.

19. Somewhere in the South Pacific: Your ship is traveling on a course 060 at a speed of 10 knots. There is a westerly ocean current with a direction of 270 and a speed of 4 knots. What is your true course and speed?

20. Somewhere over the North Atlantic: Your F18 Hornet is flying on a course 045 at a speed of 400 knots in the jet stream whose direction is 090 and a speed of 100 knots. What is your true course and ground speed?