Score:

Name:______________________________

Section (circle one): 1021 2021
Team (circle one): a b c d e

SM221 – Sample Test #2– Fall 2004

Part 1: Multiple Choice (50%). For each question, circle the letter for he best answer.

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1. Let $f(t,p)$ be the speed of sound in meters per second (mps) when the temperature is t degrees Celsius $(°C)$ and the pressure is p atmospheres (atm). The statement $\frac{\partial f}{dp}(2,10)=3$ means:

(a) At a temperature of $2°C$ and a pressure of $10\,atm$, the speed of sound is $3\,mps$.
(b) At a temperature of $2°C$ and a pressure of $10\,atm$, the speed of sound is increasing with increasing pressure at a rate of $3\,mps$ per atm.
(c) At a temperature of $2°C$ and a pressure of $10\,atm$, the pressure is at a rate of $3\,atm$ per hour.
(d) The speed of sound increases 3 mps for each $°C$ increase in temperature.
(e) If the speed of sound increases at $3\,mps$, the temperature increases at a rate of $2°C$ per hour and the pressure increase at a rate of $10\,atm$ per hour.

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2. Suppose the budget *b* in dollars of a manufacturer is a function of the number *T* of trucks and number *C* of cars produces so $b=b(T,C)$. *T* and *C* are functions of time *t*, measure in years, Suppose also that:

$$T(10)=1000,\quad C(10)=2000,\quad \frac{dT}{dt}(10)=500,\quad \frac{dC}{dt}(10)=-400,$$

and:

$$\frac{\partial b}{dT}(1000,2000)=1000,\quad \frac{\partial b}{dC}(1000,2000)=800.$$

That rate at which *b* is increasing with time when t=10, in dollars per year, is

(a) 18000 (b)1800 (c)100 (d) 0 (e) 180,000

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3. If $f(x,y)=x^2+xy$, in which direction is *f* increasing the fastest at (3,1)?

(a) $7\vec{i}+3\vec{j}$ (b) $\vec{i}+\vec{j}$ (c) $7\vec{i}-3\vec{j}$ (d) $3\vec{i}-7\vec{j}$ (e) equally fast in all directions

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4. Which of the following is a parameterization of the cylinder $x^2+y^2=16$?

(a) $r=4$ (b) $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ (c) $\vec{r}=4\cos\theta\vec{i}+4\sin\theta\vec{j}+z\vec{k}$
(d) $\vec{r}=x\vec{i}+y\vec{j}+(x^2+y^2)\vec{k}$ (e) $r^2=x^2+y^2$

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5. The contour map for the function $f(x,y)=\sqrt{x^2+y^2}$ is a family of :

(a) parabaloids (b) cones (c) circles (d) right triangles (e) spheres

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6. An equation of the tangent plane to the surface $z = e^{2x} + \cos(y)$ at the point (0,0,2) is:

(a) $z=2x+2$ (b) $x+y+z=1$ (c) $z=3x+2y$ (d) $x-y+z=0$ (e) $z=y$

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7. Suppose $\vec{r}'(t) = \langle 4e^{2t}, 4t, \cos(t) \rangle$ and $\vec{r}(0) = \langle 2,1,0 \rangle$, then $\vec{r}(1)$ equals:

(a) $\langle 2e^2, 3, \sin(1) \rangle$ (b) $\langle 2e^2+2, 2, 0 \rangle$ (c) $\langle 2e^2, 2, \sin(1) \rangle$ (d) $\langle 8e^2, 3, \sin(1) \rangle$ (e) $\langle 8e^2, 4, -\sin(1) \rangle$

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8. If $f(x,y) = x^2 y - y^2$, then $f_{xy}(2,3)$ equals,

(a) 0 (b) 2 (c) 3 (d) 4 (e) 6

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9. A particle's position is given by $\vec{r}(t) = \langle \sin(2t), e^t - 1, t^2 \rangle$. The particles speed at time $t=0$ is:

(a) 0 (b) 2 (c) $\sqrt{5}$ (d) $\sqrt{1+e^2}$ (e) $\sqrt{4+e^2}$

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10. The tangent plane to the ellipsoid $x^2 + 2y^2 + 3z^2 = 36$ at the point (1,-2,3) is:

(a) $x-4y+9z=0$ (b) $x+4y+9z=20$ (c) $x-4y+9z=20$ (d) $x-4y+9z=36$ (e) $x+4y+9z=108$

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11. Find a vector in the direction of greatest increase of the function $f(x,y,z) = x^2 - 2xy + z^2$ at the point (1,1,2).

(a) $\langle 4, -2, 2 \rangle$ (b) $\langle 4, 2, 2 \rangle$ (c) $\langle 2, -2, 4 \rangle$ (d) $\langle 0, -2, 4 \rangle$ (e) $\langle 1, 1, 2 \rangle$

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12. If $\nabla f(x_0, y_0) = \langle 3, -2 \rangle$, then the direction derivative of f, $D_{\vec{v}}(x_0, y_0)$ in the direction of vector $\vec{v} = \langle 4,3 \rangle$ is:

(a) -6 (b) $-\frac{6}{5}$ (c) 0 (d) $\frac{6}{5}$ (e) 6

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13. If $\vec{r}(t) = \left\langle t, \frac{1}{\sqrt{2}} t^2, \frac{t^3}{3} \right\rangle$, then the length of the curve between $t=0$ and $t=1$ is:

(a) 0 (b) 4/3 (c) 7/3 (d) 10/3 (e) 4

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14. If $f = f(x,y)$ is a differentiable function with partial derivatives $f_x(1,2) = 5$ and $f_y(1,2) = -2$, then the directional derivative of f at (1,2) in the direction of the vector $-3\vec{i} + 4\vec{j}$ is:

(a) -23 (b) $-15\vec{i} + -8\vec{j}$ (c) -23/5 (d) $-3\vec{i} - \frac{8}{5}\vec{j}$ (e) $6\vec{i} + 20\vec{j}$

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15. The maximum rate of change of the function $f(x, y) = x^2 y$ at the point (1,2,2) is

(a) 3 (b) 5 (c) 4 (d) $\sqrt{5}$ (e) $\sqrt{17}$

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16. If $\bar{r}(t) = \langle t, t^2, t^3 \rangle$ then a tangent vector to the curve at the point (1,1,1) is:

(a) $\langle 0, 1, 1 \rangle$ (b) $\langle 1, 1, 1 \rangle$ (c) $\langle 0, 2, 3 \rangle$ (d) $\langle 1, 2, 3 \rangle$ (e) $\langle 0, 2, 6 \rangle$

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17. The length of the curve $\langle 2t, \sin(t), \cos(t) \rangle$ for $0 \le t \le \pi$ is closest to:

(a) 2.24 (b) 3.14 (c) 6.59 (d) 7.02 (e) 10.63

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18. The position of a particle is $\bar{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$. Its acceleration when $t=3$ is $a(3)=$

(a) $2\vec{j} + 18\vec{k}$ (b) $\vec{i} + 6\vec{j} - 27\vec{k}$ (c) $3\vec{i} + 9\vec{j} - 27\vec{k}$

(d) $4.5\vec{i} + 6.75\vec{j} - 12.15\vec{k}$ (e) $4.5\vec{i} + 9\vec{j} - 20.25\vec{k}$

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19. An equation for the plane tangent to the surface $z = e^{2x+y}$ at the point (0,0,1) is:

(a) $z=1$ (b) $z = 2xe^{2x+y} + ye^{2x+y} + 1$ (c) $z=x+y+1$ (d) $z=x+2y+1$ (e) $z=2x+y+1$

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20. $f(x, y, z) = x^2 y + yz$. The directional derivative of f at the point (3,-2,4) in the direction f the unit vector $\left\langle \frac{2}{3}, \frac{2}{3}, -\frac{1}{3} \right\rangle$ is

(a) -26 (b) -2/3 (c) 0 (d) 4/3 (e) 46/9

Part 2: Free Response (50 %). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.

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21. A baseball player hits a baseball when it is 1 foot above home plate. He imparts an initial speed of 120 feet per second to the baseball, and it leaves his bat at an angle of 30° above the horizontal. Neglecting air resistance and assuming the acceleration due to gravity is 32 feet per second per second, will the baseball pass above a 12 foot high fence 360 feet away.

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22. Find the equation for the plane tangent to the surface $x = u^2 + v^2$, $y = v$, $z = u^2 - v^2$ at the point where $u=2$ and $v=-1$.

23. The function f is differentiable at (1,2). $\vec{u}$ is the unit vector $\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$ and $\vec{v}$ is the unit vector $\left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$. The direction derivative $D_{\vec{u}}(1,2) = 1$ and $D_{\vec{v}}(1,2) = -1$. Compute the gradient vector $\nabla f(1,2)$.

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24. The position of a particle in the plane is given by $\vec{r}(t) = \langle 6\cos(t/2), 8\sin(t/2) \rangle$ for $0 \le t \le 2\pi$.

(a) Compute the position vector $\vec{r}(\pi)$.

(b) Compute the position vector $\vec{r}'(\pi)$.

(c) Compute the position vector $\vec{r}''(\pi)$.

(d) Compute the speed at $t = \pi$.

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25. A projectile is fired from the top of a cliff that is 55 meters high over the level sea on a planet where the acceleration due to gravity is 10 *m/sec*^{2} (directed downward). The initial speed of the projectile is 100 *m/sec*, and the angle of inclination is $30°$ above horizontal.

(a) Show that the projectile is airborne for 11.0 *seconds* befor hitting the sea.
(b) What is the speed of the projectile at impact?

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26. Monthly production P in a coal mine is given by $P(K,L) = 8K^{1/2}L^{1/2}$, where P is measured in tons, and K in thousands of dollars spent on equipment, and L in thousands of dollars spent on labor per month. Currently *K=100* and *L=25*.

(a) Compute the current production *P(100,25)*.
(b) A computation shows the $P_L(100,25) = 8$. In words this means: "*For every $1000 more spent on ...*". Complete the sentence carefully.
(c) Compute the partial derivative $P_K(100,25)$.
(d) Suppose that the amounts spent on equipment and labor are increasing by $4000 and $3000 a month, respectively. At what rate is production increasing.

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27. A stone is swung around on a string so that the position (as measured in *meters*) of the stone at time t (in *seconds*) is $\vec{r}(t) = 2\cos(t)\vec{i} + (5 + 2\sin(t))\vec{k}$.

(a) Find the velocity, acceleration and speed of the stone at *t=0*.
(b) At $t = \frac{5\pi}{3}$ *seconds*, the string breaks and the stone is only subject to gravity. Find the position $\vec{r}(u)$ of the stone as a function of the number of seconds $u = t - \frac{5\pi}{3}$ after the string breaks, for $u > 0$. (Hint: $g = 9.8 m/s^2$)

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28. A charged particle moving in the plane receives acceleration due to an alternating electric field $\vec{a}(t) = 50\sin(\pi t/60)\vec{j}$ *m/sec*^{2}. If the initial velocity is $\vec{v}(0) = 100\vec{i}$ *m/sec* and the initial position is $\vec{r}(0) = 0\ m$,

1. Find the position $\vec{r}(t)$.
2. Determine the height at which the particle hits a screen 20 meters down the x axis.

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29. If $z = x^2 y,\ x = t^2 + 1,\ y = t + 1$, use the chain rule to find $\left.\dfrac{dz}{dt}\right|_{t=2}$

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30. The table on the right shows the actual normal take off distance, *D*, of an A6 Intruder which weighs 35000 *lbs* with a head wind of 20 *knots* as a function of temperature, T, in *degrees F* and altitude, A, in *feet*.

Temp/Alt	0	2000	4000	6000
0	940	1110	1350	1580
30	1070	1280	1530	1830
60	1280	1510	1810	2140
90	1520	1800	2190	2600

(a) Estimate $\dfrac{\partial D}{\partial T}$ when the temperature is $30°$ and the altitude is 2000 *ft*.

(b) Estimate $\dfrac{\partial D}{\partial A}$ when the temperature is $30°$ and the altitude is 2000 *ft*.

(c) Use these estimates to determine the take off distance if the temperature is $35°$ and the altitude is 2500 *ft*.