Score:

Name:______________________________

Section (circle one): 1021 2021
Team (circle one): a b c d e

SM221 – Sample Test #3– Fall 2004

Part 1: Multiple Choice (50%). For each question, circle the letter for he best answer.

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1. The volume under the paraboloid $z = x^2 + 2y^2$ and above the region in the xy-plane bounded by the x-axis and y-axis, and the line $x+y=1$ is:

(a) $\frac{1}{4}$ (b) $\frac{5}{12}$ (c) $\frac{7}{12}$ (d) $\frac{3}{4}$ (e) 1

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2. The volume above the paraboloid $z = x^2 + y^2$ and below the paraboloid $z = 1 - x^2 - y^2$ is:

(a) 0 (b) $\frac{3}{16}\pi$ (c) $\frac{1}{4}\pi$ (d) $\frac{2}{3}\pi$ (e) $\frac{2\sqrt{2}}{3}\pi$

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3. The iterated integral $\int_{-2}^{0}\int_{0}^{-x} f(x,y)\,dydx$ must be equal to:

(a) $\int_{0}^{-x}\int_{-2}^{0} f(x,y)\,dxdy$ (b) $\int_{0}^{2}\int_{-y}^{-2} f(x,y)\,dxdy$ (c) $\int_{-2}^{0}\int_{0}^{-y} f(x,y)\,dxdy$

(d) $\int_{0}^{2}\int_{-2}^{-y} f(x,y)\,dxdy$ (e) $\int_{-2}^{0}\int_{-y}^{0} f(x,y)\,dxdy$

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4. The surface area of the portion of the plane $z=2x+3y$ which lies above the rectangle $0 \le x \le 2$ and $0 \le y \le 1$ is:

(a) $2\sqrt{6}$ (b) $2\sqrt{14}$ (c) 9 (d) 12 (e) 28

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5. $\int_{0}^{\pi/2}\int_{0}^{\sin x} y\,dydx =$

(a) $\frac{1}{2}\sin^2 x$ (b) π (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{8}$ (e) 0

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6. The integral $f(x,y) = x^2 + y$ over the disk $x^2 + y^2 \le 1$ is:

(a) $\int_{0}^{1}\int_{0}^{1}(x^2+y)dxdy$ (b) $4\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}}(x^2+y)dxdy$ (c) $\int_{0}^{1}\int_{0}^{2\pi}(r^2\cos^2\theta + r\sin\theta)rd\theta dr$

(d) *0* (e) $\int_{0}^{1}\int_{0}^{2\pi}(\cos^2\theta + \sin\theta)\,d\theta dr$

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7. Let R be the region of a 3-space bounded by the xy-plane, the surface $x^2 + y^2 = 1$, and the plane $y+z=1$. Then $\iiint_R z dV =$

(a) $\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} z\,dxdydz$ (b) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{1-y} z\,dzdydx$ (c) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(1-y)\,dydx$

(d) $\int_{0}^{1}\int_{-1}^{1}\int_{-1}^{1-y} z\,dxdydz$ (e) $\int_{0}^{1}\int_{0}^{2\pi}(1-r\sin\theta)rdrd\theta$

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8. If we reverse the order of integration in the integral $\int_0^1 \int_0^{x^2} \sin(y)\, dydx$, we obtain:

(a) $\int_0^1 \int_{\sqrt{y}}^1 \sin(y)\, dxdy$ (b) $\int_0^1 \int_{\sqrt{y}}^1 \sin^{-1}(x)\, dxdy$ (c) $\int_0^1 \int_{y^2}^0 \sin(y)\, dxdy$ (d) $\int_0^1 \int_0^{y^2} \sin(y)\, dxdy$

(e) $\int_0^1 \int_x^1 \cos(y)\, dxdy$

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9. When we convert the iterated integral $\int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\, dydx$ from rectangular to polar coordinates, we obtain:

(a) $\int_0^{\pi/2} \int_0^3 e^{r^2}\, drd\theta$ (b) $\int_0^{2\pi} \int_0^3 e^{r^2}\, drd\theta$ (c) $\int_0^{\pi/2} \int_0^3 re^{r^2}\, drd\theta$ (d) $\int_0^3 \int_0^{\sqrt{9-(r\cos\theta)^2}} e^{r^2}\, rdrd\theta$

(e) $\int_0^3 \int_0^{\sqrt{9-(r\cos\theta)^2}} e^{r^2}\, drd\theta$

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10. Let R be the rectangular region with vertices (0,0), (1,0), (1,3), and (0,3). Then $\iint_R 223y^2 dA =$

(a) 0 (b) 3 (c) 223 (d) 669 (e) 2007

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11. The region of integration for the iterated integral $\int_0^4 \int_0^{6-x} (x^2 + y^2) dydx$ is a:

(a) quarter-circular disk (b) circular disk (c) trapezoid (d) right triangle (e) rectangle

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12. The iterated integral $\int_0^1 \int_0^x f(x,y)\, dydx$ is equivalent to:

(a) $\int_0^1 \int_y^1 f(x,y)\, dxdy$ (b) $\int_0^1 \int_1^y f(x,y)\, dxdy$ (c) $\int_0^1 \int_0^1 f(x,y)\, dxdy$ (d) $\int_0^1 \int_0^y f(x,y)\, dxdy$

(e) $\int_0^{\pi/4} \int_0^1 f(r\cos\theta, r\sin\theta)\, rdrd\theta$

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13. Evaluate $\iiint_E (x^2 + y^2 + z^2) dV$ where E is the solid unit sphere centered at the origin.

(a) $\frac{2}{5}\pi$ (b) $\frac{2}{3}\pi$ (c) $\frac{4}{5}\pi$ (d) π (e) $\frac{4}{3}\pi$

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14. A solid is bounded above by the surface $z = x^2 + y^2$ and below by the region in the *xy*-plane bounded by $y=0$ and $y = 1 - x^2$. If the density is given by $\rho(x,y,z) = z$, the mass of the solid is best computed with:

(a) A double integral in rectangular coordinates.
(b) A double integral in polar coordinates.
(c) A triple integral in rectangular coordinates.
(d) A triple integral in cylindrical coordinates.
(e) A triple integral in spherical coordinates.

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15. The volume of the solid below the surface $z = 16 - x^2 - y^2$ and above the *xy*-plane is:

(a) $-\frac{128}{3}\pi$ (b) $\frac{128}{3}\pi$ (c) $\frac{256}{3}\pi$ (d) 128π (e) 256π

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Part 2: Free Response (50 %). The remaining problems are not multiple choice. Answer them in the space below the problem. Show the details of your work and clearly indicate your answers.
(Challenge Factor: *Easy, ***Challenging, *****Very Challenging)

16. (****) For the double integral $\int_0^1 \int_x^{1+3x} xy\,dydx$,
   (b) Make a careful sketch of the region of integration, labeling the coordinates of all corners.
   (c) Rewrite the integral for the opposite order of the variables.

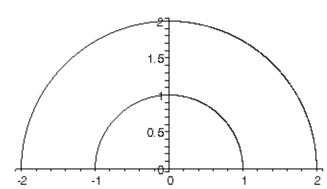
17. (**) Let R be the portion of the xy-plane with $1 \le x^2 + y^2 \le 9$.
   (b) Sketch R, and use your sketch to explain why it is sensible to set up an integral over R in a coordinate system that is not Cartesian.
   (c) Compute $\iint_R e^{x^2+y^2}\,dA$.

18. (***) For the triple integral $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{\sqrt{4-x^2-y^2}} \left(x^2 + y^2 + z^2\right) dzdydx$

   (a) Draw the region of integration.
   (b) Rewrite the integral in cylindrical coordinates.
   (c) Rewrite the integral in spherical coordinates.

19. (**) Determine the volume of a tetrahedron bounded the planes *x=0, y=0, z=0* and *3x+2y+z=6.*

20. (**) Find $\iint_D \sqrt{x^2 + y^2}\, dA$ where D is the region shown:



21. (***) Find the volume of the solid bounded above by the plane *z=x+y* and below by the region in the *xy* plane bounded by *y=0* and $y = 2x - x^2$.

22. (***) Find the mass of a solid cylinder of radius 5 and height 3, with its base sitting on the *xy*-plane with the center of the base at the origin if the density is given by $\rho(x, y, z) = x^2 + y^2$.

23. (***) A lamina (a two dimensional object) occupies the region in the *xy*-plane bounded by the curves $y=x$ and $y = x^2$. Find the center of mass of the lamina if the density at the point (x,y) is $\rho(x,y) = 2y$.

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24. (***) Find the mass of the tetrahedron bound by the coordinate planes and the plane $3x+2y+z=6$, if the density at the point (x,y,z) is $\rho(x,y,z) = x^2 + z$. <u>Set up the integral, then solve with your calculator!!!</u>

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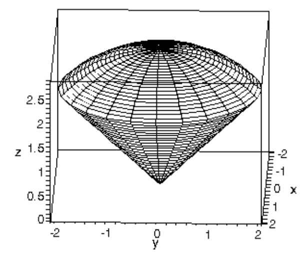
25. (*****) E is the solid bounded below by the cone $z^2 = x^2 + y^2$ and bounded above by the sphere $x^2 + y^2 + z^2 = 8$. The density at the point (x,y,z) is $\rho(x,y,z) = z$.



(b) Set up an iterated integral in cylindrical coordinates which gives the mass of *E*.
(c) Set up an iterated integral in spherical coordinates which gives the mass of *E*.
(d) Evaluate at least one of the integrals.
