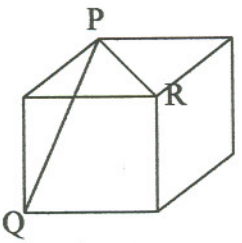
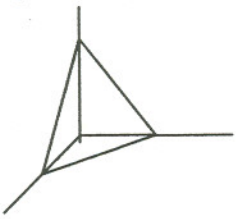
SM221 Calculus III Practice Problems I Fall 2009

1. A 1x1x1 cube has corners P, Q, and R as shown on the right. Use vectors to find the angle between $\overline{PQ}$ and $\overline{PR}$.



2. a) Find an equation for the plane on the right which intersects the x-axis at 2, the y-axis at 3, and the z-axis at 6.
   b) What is the area of the triangle shown?



3. Let $\vec{v} =< 1, 3 >$ and $\vec{w} =< 4, 2 >$.
   a) Find the scalar $comp_{\vec{w}}\,\vec{v}$. b) Find the vector $proj_{\vec{w}}\,\vec{v}$.
   c) Draw $\vec{v}$, $\vec{w}$, and $proj_{\vec{w}}\,\vec{v}$ on the same set of axes.

4. Consider the two points A(1,2,3) and B(4,0,4).
   (a) Write parametric equations for the line going through the points A and B.
   (b) Where does this line intersect the y-z plane?
   (c) Is your line in part (a) parallel to the plane $3x - 2y + z = 2$? Why?

5. Consider the curve in R^3 given parametrically by $x = \cos(t)$, $y = t$, $z = \sin(t)$.
   (a) Sketch by hand the curve in R^3 (3 dimensional space).
   (b) Find the position vector, velocity vector, speed, and acceleration vector at time t ($\vec{r}(t)$, $\vec{v}(t)$, *speed*(t), *and* $\vec{a}(t)$).
   (c) Find $\vec{r}(t)$, $\vec{v}(t)$, *speed*(t), *and* $\vec{a}(t)$ at time $t = \pi/4$ and plot the vectors on your graph from part (a).
   (d) Find parametric equations for the line tangent to your curve at the point corresponding to $t = \pi/4$.
   (e) Find the arc length of the curve as $t : 0 \to 2\pi$.

6. State whether each of the following is a vector, a scalar, or makes no sense:

   (a) $(\vec{a}\cdot\vec{b})\vec{c}$ (b) $\dfrac{\vec{c}}{\vec{a}\cdot\vec{b}}$ (c) $\dfrac{\vec{a}\cdot\vec{b}}{\vec{c}}$ (d) $(\vec{a}\times\vec{b})\cdot|\vec{c}|$

   (e) $(\vec{a}\times\vec{b})\times\vec{c}$ (f) $\vec{a}\cdot(\vec{b}\times\vec{c})$ (g) $(\vec{a}\cdot\vec{b})\times\vec{c}$ (h) $(\vec{a}\cdot\vec{b})+\vec{c}$.

7. Sketch for each of the following functions a graphs in R^3:

   (a) $\vec{r}(t) =< t\cos(t), t\sin(t), t >$, (b) $z = y^2$, (c) $z = x^2 + \dfrac{y^2}{4}$.

8. Astronaut Allan Shepherd once hit a golf ball from the surface of the moon. If the ball left his club going 200 ft/sec and made an angle of 45° with the horizontal and gravity on the moon is only 1/6 of that on earth, how far did the golf ball go (horizontal distance in yards)? Start with accel $\vec{a}(t)$ and show all of your work.