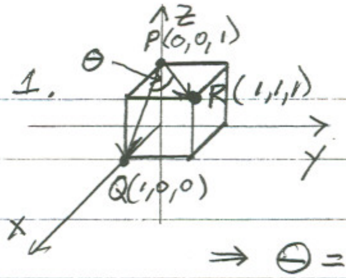
SM 221 PRACTICE PROBLEMS I SOLUTIONS FALL 2009

1.



$\overrightarrow{PQ} = \langle 1,0,-1\rangle$

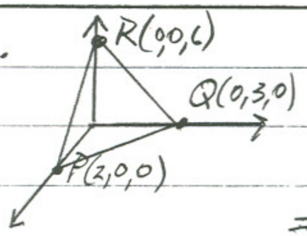
$\overrightarrow{PR} = \langle 1,1,0\rangle$

$\overrightarrow{PQ}\cdot\overrightarrow{PR} = |\overrightarrow{PQ}||\overrightarrow{PR}|\cos(\theta)$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}\right)$

$= \cos^{-1}\left(\dfrac{1}{\sqrt{2}\sqrt{2}}\right) = 60^\circ$

2.



(a) normal vector

$\vec{N} = \overrightarrow{PQ}\times\overrightarrow{PR}$

$= \langle -2,3,0\rangle \times \langle -2,0,6\rangle$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 0 \\ -2 & 0 & 6 \end{vmatrix} = \langle 18, 12, 6\rangle$

use the pt $P(2,0,0)$ (or Q or R)

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

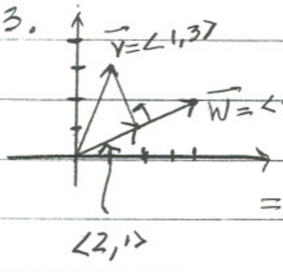
$\Leftrightarrow 18(x-2)+12(y-0)+6(z-0)=0$

$\Leftrightarrow 18x+12y+6z=36 \Leftrightarrow 3x+2y+1z=6$

(b) $\text{area}\,\Delta = \frac{1}{2}|\overrightarrow{PQ}\times\overrightarrow{PR}| = \frac{1}{2}\sqrt{18^2+12^2+6^2}$

$= \frac{1}{2}\sqrt{504} = 3\sqrt{14}$

3.



$\text{comp}_{\vec{W}}\vec{V} = \vec{V}\cdot\hat{W}$ ← unit vector

$= \langle 1,3\rangle\cdot\dfrac{\langle 4,2\rangle}{\sqrt{16+4}} = \dfrac{10}{\sqrt{20}}$.

$\text{proj}_{\vec{W}}\vec{V} = (\vec{V}\cdot\hat{W})\,\hat{W}$

$= \dfrac{10}{\sqrt{20}}\,\dfrac{\langle 4,2\rangle}{\sqrt{20}} = \langle 2,1\rangle$.

4. $\vec{N} = \overrightarrow{AB} = \langle 3,-2,1\rangle$

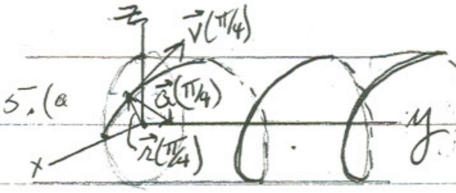
a) $x=1+3t$; $y=2-2t$; $z=3+1t$

b) y-z plane $\Rightarrow x=0 \Rightarrow 1+3t=0 \Rightarrow t=-1/3$

$\Rightarrow (x,y,z) = (0, 8/3, 8/3)$

c) the line is parallel to the vector $\vec{v}=\langle 3,-2,1\rangle = \vec{N} \perp$ the plane. So no, the line is not parallel to the plane, it is $\perp$ the plane.

5. (a)



helix around y axis

(b) $\vec{r}(t) = \langle \cos(t), t, \sin(t)\rangle$

$\vec{v}(t) = \langle -\sin(t), 1, \cos(t)\rangle$

$\text{speed}(t) = |v(t)| = \sqrt{\sin^2(t)+1+\cos^2(t)} = \sqrt{2}$

$\vec{a}(t) = \langle -\cos(t), 0, -\sin(t)\rangle$

(c) $\vec{r}(\pi/4) = \langle \frac{\sqrt{2}}{2}, \frac{\pi}{4}, \frac{\sqrt{2}}{2}\rangle$; $\vec{v}(\pi/4) = \langle -\frac{\sqrt{2}}{2}, 1, \frac{\sqrt{2}}{2}\rangle$

$\text{speed}(\pi/4) = \sqrt{2}$; $\vec{a}(\pi/4) = \langle -\sqrt{2}/2, 0, -\frac{\sqrt{2}}{2}\rangle$

$\vec{r}(\pi/4)$ = position vector; $\vec{v}(\pi/4)$ is tang to curve

$\vec{a}(\pi/4)$ is $\perp$ $\vec{v}(\pi/4)$ for this path.

d) $x = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}t$; $y = \pi/4 + 1t$; $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}t$.

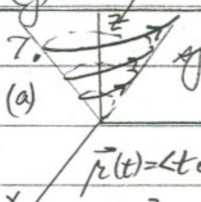
e) $\ell = \int_0^{2\pi} |\vec{r}\,'(t)|\,dt = \int_0^{2\pi}\sqrt{2}\,dt = 2\sqrt{2}\pi$

6. (a) vector; (b) vector; (c) no sense

(d) no sense; (e) vector; (f) scalar

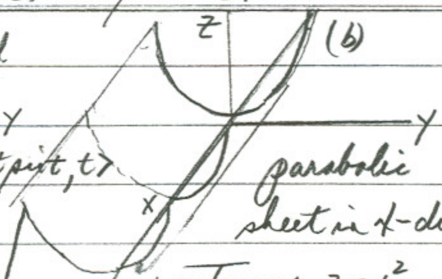
(g) no sense; (h) no sense.

7. (a)



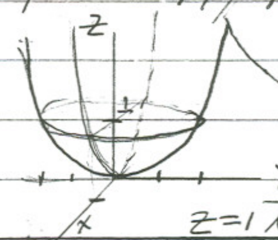
$\vec{r}(t) = \langle t\cos t, t\sin t, t\rangle$

(b)



parabolic sheet in x-dir

(c)



x-z trace: $z = x^2$

y-z trace: $z = y^2/4$

$z=1$ trace: $1 = x^2 + y^2/4$ ellipse

8

$g = \frac{32}{6}\,\text{ft/s}^2 \doteq 5.3\ \text{ft/s}^2$

$\vec{a}(t) = \langle 0, -5.3\rangle \Rightarrow \vec{v}(t) = \int \vec{a}(t)\,dt = \langle c_1, -5.3t + c_2\rangle$

$\vec{v}(0) = \langle 200\cos 45^\circ, 200\sin 45^\circ\rangle = \langle 100\sqrt{2}, 100\sqrt{2}\rangle$

$\Rightarrow c_1 = 100\sqrt{2}$; $c_2 = 100\sqrt{2} \Rightarrow v(t) = \langle 100\sqrt{2}, -5.3t + 100\sqrt{2}\rangle$

$\Rightarrow \vec{r}(t) = \int \vec{v}(t)\,dt = \langle 100\sqrt{2}\,t + c_1, -2.65t^2 + 100\sqrt{2}\,t + c_2\rangle$

$\vec{r}(0) = \langle 0,0\rangle \Rightarrow \vec{r}(t) = \langle 100\sqrt{2}\,t, -2.65t^2 + 100\sqrt{2}\,t\rangle$

$y(t) = -2.65t^2 + 100\sqrt{2}\,t = 0$ when $t=0$ or $t = 53.37$

horiz dist $= x(53.37) = 100\sqrt{2}(53.37) =$

$= 7{,}548$ ft $= 2{,}516$ yds