1. Consider the function \( f(x, y) = x^2 + \frac{y^2}{4} \).

(a) Sketch the contour curves (level sets) where \( f(x, y) = k \) for \( k = 1 \) and \( k = 4 \).
(b) Find \( \nabla f(1,2) \), (gradient of \( f \) at \((1,2)\)), and draw it on your graph in (a).
(c) Find the directional derivative for \( f \) at \((1,2)\) in the direction \( 3\hat{i} + 4\hat{j} \).
(d) What is the greatest directional derivative possible for \( f \) at \((1,2)\) ?
(e) At \((1,2)\), what directions gives zero for the directional derivative of \( f \) ?
(f) Sketch the graph of the function \( z = x^2 + \frac{y^2}{4} \) by drawing traces on the x-z plane, the y-z plane, and the planes \( z = 1 \) and \( z = 4 \).

2. Find both partial derivatives for

(a) \( f(x, y) = y^2 e^{3x} \);
(b) \( z = (3xy + 2x)^5 \);
(c) \( g(x, y) = \frac{y^2}{x+1} \).

3. Values for a function \( z = f(x, y) \) are given in the table on the right. (a) Estimate \( \frac{\partial f}{\partial x} (20,10) \). (b) Estimate \( \frac{\partial f}{\partial y} (20,10) \).
(c) Estimate the equation for the plane tangent to the surface \( z = f(x, y) \) at the point \((20,10,25)\) by using (a) and (b).
(d) Use your tangent plane to approximate \( f(21,9) \).

4. A contour diagram for a function \( z = f(x, y) \) is drawn on the right.
(a) Approximate \( f_x (3,1) \).
(b) Approximate \( f_y (3,1) \).
(c) Use (a) and (b) to approximate \( \nabla f(3,1) \) and draw it on the graph.

5. Use the chain rule to find \( \frac{\partial z}{\partial s} \) at \( s = 2 \), \( t = 1 \) if \( z = e^{x-3y} \), where \( x = s^2 t \) and \( y = st^2 \).

6. Use a gradient to find the equations for the tangent plane and the normal line at the point \((1,2,3)\) to the ellipsoid whose equation is \( \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 3 \).

7. Find the local maxima, minima points for the function
\( f(x, y) = y^2 + 6x^3 + 18x^2 - 6xy + 18x - 18y \).