SM221 CALCULUS III FALL 2009 PRACTICE PROBLEMS II

NAME:____________________________________

1. Consider the function $f(x,y) = x^2 + \dfrac{y^2}{4}$.

(a) Sketch the contour curves (level sets) where $f(x,y) = k$ for $k = 1$ and $k = 4$.

(b) Find $\vec{\nabla} f(1,2)$, (gradient of f at (1,2)), and draw it on your graph in (a).

(c) Find the directional derivative for f at (1,2) in the direction $3\hat{i} + 4\hat{j}$.

(d) What is the greatest directional derivative possible for f at (1,2)?

(e) At (1,2), what directions gives zero for the directional derivative of f ?

(f) Sketch the graph of the function $z = x^2 + \dfrac{y^2}{4}$ by drawing traces on the x-z plane, the y-z plane, and the planes z = 1 and z = 4.

2. Find both partial derivatives for

(a) $f(x,y) = y^2 e^{3x}$; (b) $z = (3xy + 2x)^5$; (c) $g(x,y) = \dfrac{y^2}{x+1}$.

3. Values for a function $z = f(x,y)$ are given in the table on the right. (a) Estimate $\dfrac{\partial f}{\partial x}(20,10)$. (b) Estimate $\dfrac{\partial f}{\partial y}(20,10)$.

(c) Estimate the equation for the plane tangent to the surface $z = f(x,y)$ at the point (20,10,25) by using (a) and (b).

(d) Use your tangent plane to approximate $f(21,9)$.

x

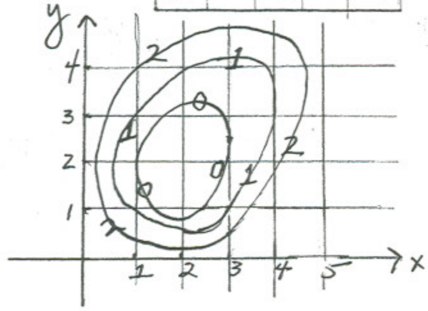
y \ x	0	10	20	30
0	12	18	26	43
10	10	16	25	36
20	7	13	21	35
30	3	9	16	29

y

4. A contour diagram for a function $z = f(x,y)$ is drawn on the right.

(a) Approximate $f_x(3,1)$. (b) Approximate $f_y(3,1)$.

(c) Use (a) and (b) to approximate $\vec{\nabla} f(3,1)$ and draw it on the graph.



5. Use the chain rule to find $\dfrac{\partial z}{\partial s}$ at $s = 2$, $t = 1$ if $z = e^{x-3y}$, where $x = s^2 t$ and $y = st^2$.

6. Use a gradient to find the equations for the tangent plane and the normal line at the point (1 , 2 , 3) to the ellipsoid whose equation is $x^2 + \dfrac{y^2}{4} + \dfrac{z^2}{9} = 3$.

7. Find the local maxima, minima points for the function
$f(x,y) = y^2 + 6x^3 + 18x^2 - 6xy + 18x - 18y$.