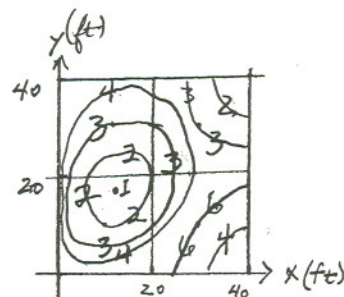


NAME: _____

1. The contour map on the right shows the depth of a square pond in feet.

(a) Use the Midpoint Rule with $m = n = 2$ to estimate the volume of the water in the pond.

(b) Estimate the average depth of the water.



2. Evaluate a double integral to find the mass of the plate bounded in the x - y plane by the curves $y = x^2$ and $y = x + 2$ if the density of the plate is given by $\rho(x, y) = y$.

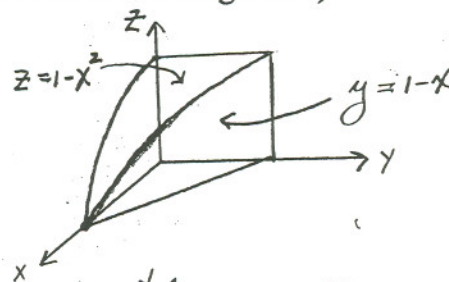
3. Evaluate by hand $\int_0^1 \int_x^1 \cos(y^2) dy dx$. (Hint: Change the order of integration.)

4. Consider the object sketched on the right.

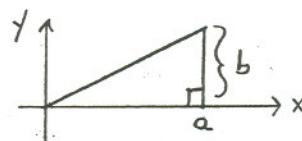
Set up (but do not evaluate) integrals for the volume

(a) of the form $\iint_R f(x, y) dy dx$, and

(b) of the form $\iiint_E f(x, y, z) dy dz dx$.



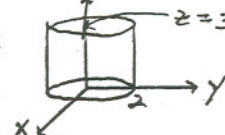
5. Use double integrals to find the center of mass (\bar{x}, \bar{y}) for the right triangle on the right if its density is constant. *Guess first.*



6. Find the mass of the region on the right bounded by semi-circles and the x -axis if the density at a point is proportional to its distance from the origin.



7. Find the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the circular cylinder on the right if the density at a point equals its distance from the x - y plane.



8. (a) Find the cylindrical (r, θ, z) , and spherical (ρ, θ, ϕ) coordinates for the point whose rectangular coordinates (x, y, z) are $(2, 2, 2)$.

(b) Sketch the solid described by $2 \leq r \leq 3$; $0 \leq \theta \leq \pi/2$; $0 \leq z \leq 4$.

(c) Sketch the solid described by $2 \leq \rho \leq 3$; $0 \leq \theta \leq \pi/2$; $\pi/2 \leq \phi \leq \pi$.

9. Prove property #11 on page 971 using properties #7, #8, and #10.

10. Set up the bounds of integration for the following triple integral over the hemisphere

(a) $\iiint_E f(\rho, \theta, \phi) \rho^2 \sin(\phi) d\rho d\phi d\theta$;

(b) $\iiint_E f(r, \theta, z) r dz dr d\theta$;

(c) $\iiint_E f(x, y, z) dz dy dx$.

