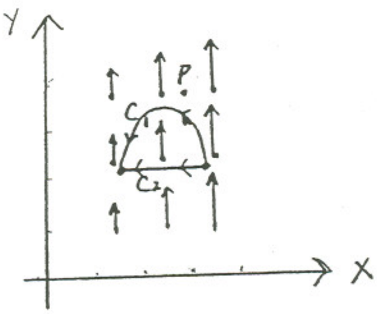
SM221 CALCULUS III FALL 2009 PRACTICE PROBLEMS IV

NAME:________________________________

1. Sketch a graph of the 2-dimensional vector field $\vec{F}(x,y) = \frac{y}{2}\hat{i} + 0\hat{j} = <\frac{y}{2}, 0>$.

2. A vector field $\vec{F}(x,y)$ is drawn on the right along with two paths $C_1$ and $C_2$ and a point P.




(a) Is div $\vec{F}$ (P) $> 0, < 0$, or $= 0$?

(b) Is $curl\,\vec{F}(P) = <0,0,0>$?

(c) Is $\int_{C_1} \vec{F}\cdot d\vec{r} > 0, < 0$, or $= 0$?

(d) Is $\int_{C_2} \vec{F}\cdot d\vec{r} > 0, < 0$, or $= 0$?

(e) Is $\vec{F}$ a conservative vector field? Why?

(f) Find a possible formula for $\vec{F}(x,y)$.

3. Prove that if $f(x,y,z)$ is any scalar field with continuous second partial derivatives, then $curl(grad(f)) = <0,0,0>$.

4. If $f(x,y,z) = \cos(x) + y^2 z$ and $\vec{F}(x,y,z) = \sin(y)\vec{i} - xz^2\vec{k}$, find the gradient, divergence, and curl of $f$ *and / or* $\vec{F}$ whenever it makes sense.

5. Rewrite each of the following expressions in terms of the "del" operator $\vec{\nabla}$, and state if each is a vector field, a scalar field, or makes no sense.
(a) div ( curl $f$ ), (b) grad ( div $\vec{F}$ ), (c) div ( grad $\vec{F}$ ), (d) (grad $f$ ) x (curl $\vec{F}$ ),
(e) div ( div $\vec{F}$ ), (f) curl ( curl $\vec{F}$ ), (g) div (curl ( grad $f$ ) ).

6. (a) State the Fundamental Theorem for Line Integrals (FTLI).
(b) Verify the FTLI for $\vec{F} = (yz)\vec{i} + (xz)\vec{j} + (xy + \cos(z))\vec{k}$, if C the line from (0,0,0) to $(1,1,\pi)$.

7. (a) State Stokes Theorem. (b) Verify Stokes Theorem for $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ where S is the part of the plane 2x + 3y + z = 6 that lies in the first octant.

8. (a) State the Divergence Thm. (b) Verify the Divergence Thm for $\vec{F} = x\vec{i} + y\vec{j} + z^2\vec{k}$ where E is the cylinder enclosed by the surfaces $z = 0$, $z = 5$, and $x^2 + y^2 = 9$.

9. (a) State Green's Theorem.
(b) If $C$ is a simple closed counter-clockwise curve in the x-y plane, which of the following line integrals does not give the area enclosed by $C$?

(i) $\oint_C x\,dy$, (ii) $\oint_C -y\,dx$; (iii) $\frac{1}{2}\oint_C x\,dy - y\,dx$; (iv) $\oint_C 3x\,dy + 2y\,dx$; (v) $\oint_C y\,dx$.