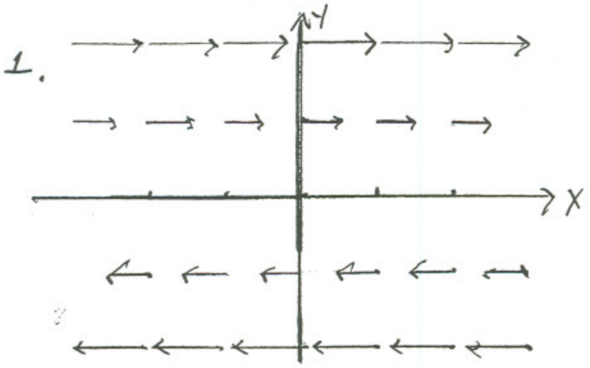
1.



2. $\vec{F} = \langle F_1, F_2, F_3 \rangle$; $\frac{\partial F_2}{\partial x} > 0$; $\frac{\partial F_2}{\partial y} = \frac{\partial F_2}{\partial z} = 0$

(a) $\operatorname{div} \vec{F}(P) = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0$

(b) $\operatorname{curl}(\vec{F}(P)) = \vec{\nabla} \times \vec{F}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_1 & F_2 & F_3 \end{vmatrix} = \hat{i}\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right) - \hat{j}\left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right) + \hat{k}\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) = a\hat{k}$ where $a > 0$, $\neq \langle 0,0,0 \rangle$

(c) $\int_{C_1} \vec{F} \cdot d\vec{r} > 0$ (d) $\int_{C_2} \vec{F} \cdot d\vec{r} = 0$ $(\vec{F} \perp d\vec{r})$

(e) $\vec{F}$ is not conservative because $\operatorname{curl}(\vec{F}) \neq \langle 0,0,0 \rangle$ and $\oint_{C = C_1 + C_2} \vec{F} \cdot d\vec{r} \neq 0$.

(f) $\vec{F}(x,y) = \frac{1}{2} x \hat{j}$ possibly.

3. see proof on p. 1063

4. $\operatorname{grad}(f) = \vec{\nabla} f = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle f$

$= \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle = \langle -\sin(x), 2yz, y^2 \rangle$.

$\operatorname{div}(f) = \vec{\nabla} \cdot f$ no sense; $\operatorname{curl}(f) = \vec{\nabla} \times f$ no sense

$\operatorname{grad}(\vec{F}) = \vec{\nabla} \vec{F}$ no sense.

$\operatorname{div}(\vec{F}) = \vec{\nabla} \cdot \vec{F} = \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \cdot \langle F_1, F_2, F_3 \rangle$

$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0 + 0 - 2xz$.

$\operatorname{curl}(\vec{F}) = \vec{\nabla} \times \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin y & 0 & -xz^2 \end{vmatrix}$

$= \langle 0, z^2, -\cos(y) \rangle$

5. (a) $\operatorname{div}(\operatorname{curl} f) = \vec{\nabla} \cdot (\vec{\nabla} \times f)$ no sense

(b) $\operatorname{grad}(\operatorname{div} \vec{F}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{F}) = $ N.f.

(c) $\operatorname{div}(\operatorname{grad} \vec{F}) = \vec{\nabla} \cdot (\vec{\nabla} \vec{F})$ no sense

(d) $(\operatorname{grad} f) \times (\operatorname{curl} F) = \vec{\nabla} f \times (\vec{\nabla} \times \vec{F}) = $ N.f.

(e) $\operatorname{div}(\operatorname{div} \vec{F}) = \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{F})$ no sense

(f) $\operatorname{curl}(\operatorname{curl} \vec{F}) = \vec{\nabla} \times (\vec{\nabla} \times \vec{F}) = $ N.f.

(g) $\operatorname{div}(\operatorname{curl}(\operatorname{grad} f)) = \vec{\nabla} \cdot (\vec{\nabla} \times (\vec{\nabla} f)) = $ A.f.

6. (a) $\int_C \vec{F} \cdot d\vec{r} = f\Big|_A^B$ if $\vec{F} = \vec{\nabla} f$.

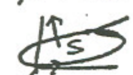
(b) $\vec{\nabla} \times \vec{F} = \vec{0} \Rightarrow \vec{F}$ is conservative $\Rightarrow \vec{F} = \nabla f$ for $f(x,y,z) = xyz + \sin(z)$

$f\Big|_A^B = xyz + \sin(z)\Big|_{(0,0,0)}^{(1,1,\pi)} = (1 \cdot 1 \cdot \pi + \sin(\pi)) - (0 + \sin(0)) = \pi$.

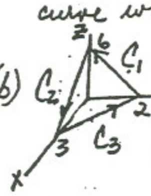
$\int_C \vec{F} \cdot d\vec{r}$ $[\vec{r} = \langle t, t, \pi t \rangle, t: 0 \to 1; d\vec{r} = \langle 1, 1, \pi \rangle dt] = \int_0^1 \langle t \cdot \pi t, t \cdot \pi t, t \cdot t + \cos(\pi t) \rangle \cdot \langle 1, 1, \pi \rangle dt$

$= \int_0^1 [\pi t^2 + \pi t^2 + \pi t^2 + \pi \cos(\pi t)] dt = \pi t^3 + \sin(\pi t)\Big|_0^1 = \pi$. ✓

7. (a) $\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$ where C is a single, closed curve which is the boundary of the surface S.



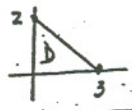
(b)



$\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S}$; $\vec{r}(x,y) = \langle x, y, 6 - 2x - 3y \rangle$

$d\vec{S} = \vec{r}_x \times \vec{r}_y \, dy\,dx = \begin{vmatrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -3 \end{vmatrix} dy\,dx = \langle 2, 3, 1 \rangle dy\,dx$ upward orientation

$= \iint_D \langle 1,1,1 \rangle \cdot \langle 2,3,1 \rangle dy\,dx = 6 \iint_D dy\,dx = 6 \cdot \text{area} = 6 \cdot 6/2 = 18$



$\int_{C_1} \vec{F} \cdot d\vec{r}$: $\vec{r}(t) = \langle 0, 2 - 2t, 6t \rangle$: $t: 0 \to 1$, $d\vec{r} = \langle 0, -2, 6 \rangle dt$

$= \int_0^1 \langle 6t, 0, 2 - 2t \rangle \cdot \langle 0, -2, 6 \rangle dt = \int_0^1 (12 - 12t) dt = 12t - 6t^2\Big|_0^1 = 6$

$\int_{C_2} \vec{F} \cdot d\vec{r}$: $\vec{r}(t) = \langle 3t, 0, 6 - 6t \rangle$; $d\vec{r} = \langle 3, 0, -6 \rangle dt$; $t: 0 \to 1$

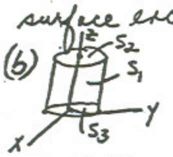
$= \int_0^1 \langle 6 - 6t, 3t, 0 \rangle \cdot \langle 3, 0, -6 \rangle dt = \int_0^1 (18 - 18t) dt = 9$

$\int_{C_3} \vec{F} \cdot d\vec{r}$: $\vec{r}(t) = \langle 3 - 3t, 2t, 0 \rangle$; $t: 0 \to 1$; $d\vec{r} = \langle -3, 2, 0 \rangle dt$

$= \int_0^1 \langle 0, 3 - 3t, 2t \rangle \cdot \langle -3, 2, 0 \rangle dt = \int_0^1 (6 - 6t) dt = 3$

so $\oint_C \vec{F} \cdot d\vec{r} = 6 + 9 + 3 = 18$ ✓

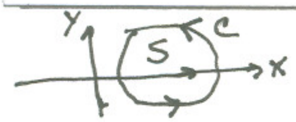
8. (a) $\iint_S \vec{F}\cdot d\vec{S} = \iiint_E (\text{div}\,\vec{F})\,dV$ where S is the surface enclosing the object E. S is oriented outward.

(b) 

$\iiint_E \vec{\nabla}\cdot\vec{F}\,dV = \iiint (1+1+2z)\,dV = \int_0^{2\pi}\int_0^3\int_0^5 (2+2z)\,dz\,r\,dr\,d\theta$
$= (2z+z^2)\Big|_0^5 \int_0^{2\pi}\int_0^3 r\,dr\,d\theta = 35\frac{r^2}{2}\Big|_0^3\int_0^{2\pi} d\theta = \underline{315\pi}$

$\iint_{S_1} \vec{F}\cdot d\vec{S}$: $x = 3\cos\theta,\ y = 3\sin\theta,\ z = z$; $r(\theta,z) = \langle 3\cos\theta, 3\sin\theta, z\rangle$
$d\vec{S} = \vec{r}_\theta \times \vec{r}_z\,d\theta\,dz = \begin{vmatrix} i & j & k \\ -3\sin\theta & 3\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} d\theta\,dz = \langle 3\cos\theta, 3\sin\theta, 0\rangle\,d\theta\,dz$
$= \iint \langle 3\cos\theta, 3\sin\theta, z^2\rangle\cdot\langle 3\cos\theta, 3\sin\theta, 0\rangle\,d\theta\,dz = \int_0^5\int_0^{2\pi} 9\,d\theta\,dz = 90\pi$
$\iint_{S_3} \vec{F}\cdot d\vec{S} = \iint \langle x,y,z^2\rangle\cdot\langle 0,0,-1\rangle\,dx\,dy = \iint_{S_3} -\overset{0}{z^2}\,dx\,dy = 0$
$\iint_{S_2} \vec{F}\cdot d\vec{S} = \iint_{S_2} \langle x,y,z^2\rangle\langle 0,0,1\rangle\,dx\,dy = \iint z^2\,dx\,dy = 25\,\text{area} = 225\pi$

$90\pi + 0 + 225\pi = \underline{315\pi}$ ✓



Let C be a simple closed curve oriented counterclockwise in the x-y plane. Let S be the region bounded by C.

9. (a) then $\oint_C P\,dx + Q\,dy = \iint_S \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$.

(b) all $= \iint_S 1\,dx\,dy$ = area enclosed
except for (v) $\int y\,dx = \iint_S (0-1)\,dx\,dy$
$= -$area.