SM221 Practice Test 4 (Melles)

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Part I – No calculators allowed.

1. Derive the formula for the area element $dA = r\,dr\,d\theta$ for double integrals in polar coordinates.

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Part II –Calculators are allowed for this part of the test.

2. Identify the surface given by each parametrization.

a. $\vec{r}(x, y) = \langle x, y, 3 - x - y \rangle$

b. $\vec{r}(u, v) = \langle u, v, 3u^2 + 3v^2 \rangle$

c. $\vec{r}(\theta, z) = \langle 3\cos\theta, 3\sin\theta, z \rangle$

d. $\vec{r}(\varphi, \theta) = \langle 3\sin\varphi\cos\theta, 3\sin\varphi\sin\theta, 3\cos\varphi \rangle$

e. $\vec{r}(r, \theta) = \langle 3r\ \cos\theta, 3r\sin\theta, 3r \rangle$

3. Use the Divergence Theorem to find the flux $\iint_S \boldsymbol{F} \cdot \boldsymbol{dS}$, where S is the surface of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 0$ and $x = 3$, with the outward orientation, and $\boldsymbol{F}(x, y, z) = y\boldsymbol{i} + x\boldsymbol{j} + 5z\boldsymbol{k}$.

4. Use Stokes' Theorem to find $\iint_S \text{curl}\, \boldsymbol{F} \cdot \boldsymbol{dS}$, where $\boldsymbol{F}(x, y, z) = 3y\boldsymbol{i} + 5x\boldsymbol{j} + 4y\boldsymbol{k}$ and S is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$, with the upward orientation. Be sure to show clearly how Stokes' Theorem was used.

5. Evaluate the surface integral $\iint_S \boldsymbol{F} \cdot \boldsymbol{dS}$, where $\boldsymbol{F}(x, y, z) = 4x\boldsymbol{i} + z\boldsymbol{j} + 5\boldsymbol{k}$ and S is the cylinder given by the parametrization $\boldsymbol{r}(u, v) = \langle 2\cos v, u, 2\sin v \rangle$, for $0 \leq u \leq 3$ and $0 \leq v \leq 2\pi$ with the outward orientation.

6. Evaluate the surface integral $\iint_S \sqrt{1 + 4y}\, dS$, where S is the part of the paraboloid $y = x^2 + z^2$ such that $x^2 + z^2 \leq 1$.