Instructions:
1. You have three hours to complete this test.
2. Fill out the Scantron form
   a. Bubble in your Alpha Number
   b. Bubble in Version 0 on the top line of the Version section
   c. Write your name and section in the top right of the form
3. Read each problem carefully. Ask questions if you do not understand the problem.
4. Read through the exam first. Start with the easy problems. You may complete the exam in any order.
5. Record all Multiple Choice answers on the Scantron form.
6. SHOW ALL YOUR WORK – partial credit will be awarded for correct work.
7. Calculators are NOT ALLOWED for use on any part of this test

HONOR COMMITMENT

By signing below, I certify that:
1. the work that I will write in this exam represents my own work and my best understanding of the material; and
2. I will neither give assistance to another student nor receive assistance during this exam except that which comes from my instructor.

__________________________________________
(signature)
1. Which of the following points is on the circle \((x - 3)^2 + (y + 1)^2 = 25\)
   a. \((3, -1)\)
   b. \((-1, 2)\)
   c. \((-2, 0)\)
   d. \((-2, 4)\)
   e. \((-3, 1)\)

2. Which number is closest to the value of \(x = \log_3(20)\)?
   a. 0.15
   b. 6.7
   c. 59
   d. 2.7
   e. 7999

3. Which line represents the inverse function of \(y = 3x - 7\)?
   a. \(y = -\frac{1}{3}x + 7\)
   b. \(y = -3x + 7\)
   c. \(y = \frac{1}{3}x - \frac{1}{7}\)
   d. \(y = -\frac{1}{3}x + \frac{7}{3}\)
   e. \(y = \frac{1}{3}x + \frac{7}{3}\)

4. Which graph represents \(y = \log(x + 2) - 1\)?
5. If \(3x + 5 = 7\), which of the following is true
   \[3x = 2\]
   \[x = \frac{2}{3}\]
   \[(1000)^{\frac{2}{3}} = (10^3)^{\frac{2}{3}} = 10^{3 \cdot \frac{2}{3}} = 10^2 = 100\]
   a. \(9x^2 + 25 = 49\)
   b. \(3 \ln(x) = \ln(2)\)
   c. \(100^x = 100\)
   d. \(18x + 5 = 42\)
   e. None of the above

6. Which of the following is equal to \(\sqrt{2(9 + 36)}\)
   a. \(3\sqrt{10}\)
   b. \(9\sqrt{2}\)
   c. \(\sqrt{18} + \sqrt{72}\)
   d. \(45\)
   e. \(6\sqrt{5}\)

7. Which is the domain of the following function, \(f(x) = \frac{\ln(9-x)}{9+x}\)
   a. \((-\infty, \infty)\)
   b. \((-\infty, -9) \cup [9, \infty)\)
   c. \((-\infty, -9) \cup (-9,9) \cup (9,\infty)\)
   d. \((-\infty, -9] \cup [9, \infty)\)
   e. \((-\infty, -9) \cup (-9,9)\)

8. Which of the following is a line through the point \((-7,2)\) and parallel to \(-7x + 3y = 61\)?
   a. \(y = \frac{3}{7}x - \frac{2}{7}\)
   b. \(y = \frac{7}{3}x + \frac{61}{3}\)
   c. \(y = \frac{7}{3}x + \frac{55}{3}\)
   d. \(y = -\frac{7}{3}x - \frac{55}{3}\)
   e. None of the above
9. Solve the equation \((5x - 5)^{1/3} + 10 = 13\)
   - a. \(\frac{5}{32}\)
   - b. \(\frac{32}{5}\)
   - c. \(\frac{27}{5}\)
   - d. \(-\frac{27}{5}\)
   - e. none of the above

10. Solve the equation \(3^{(6-3x)} = \frac{1}{27}\)
    - a. 3
    - b. -3
    - c. 9
    - d. \(\frac{1}{9}\)
    - e. 0

11. Given the following equation, \(10^{0.9542} = 9\), which equation below is true?
    - a. \(9 = \log_{10}(0.9542)\)
    - b. \(0.9542 = \log_{10}(10)\)
    - c. \(10 = \log_{10}(0.9542)\)
    - d. \(0.9542 = \log(9)\)
    - e. None of the above

12. Find the value of \(\log_{12}(6) + \log_{12}(24)\)
    - a. \(\frac{5}{2}\)
    - b. 2
    - c. \(\frac{1}{2}\)
    - d. \(\frac{1}{3}\)
    - e. 3
13. Write the following expression in condensed form

-3[4 \ln(x - 4) - \ln(x + 4)]

a. $-12 \ln \left(\frac{(x-4)^4}{(x+4)}\right)$

b. $\ln \left(\frac{(x+4)^3}{(x-4)^2}\right)$

c. $\ln \left(\frac{(x+4)^4}{(x-4)^2}\right)$

d. $\ln \left(\frac{3}{(x+4)^4}\right)$

e. $\ln \left(\frac{3}{(x-4)^4(x+4)}\right)$

14. What is the y-intercept of the following piecewise function?

$$y = f(x) = \begin{cases} x^2 + 1, & x < 1 \\ -(x + 3), & x \geq 1 \end{cases}$$

a. $(0, -3)$

b. $(-3,0)$

c. $(0,1)$

d. $(-1,0)$

e. There is no y-intercept

15. Find the angle below that is co-terminal with $\theta = -\frac{61\pi}{6}$ (i.e. points in the same direction)

a. $\frac{\pi}{6}$

b. $\frac{49\pi}{6}$

c. $\frac{5\pi}{6}$

d. $-\frac{\pi}{6}$

e. None of the above

$$-\frac{61\pi}{6} + 2\pi = \frac{-61\pi}{6} + \frac{12\pi}{6} = -\frac{49\pi}{6}$$

and so on

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16. If the side opposite angle, $\theta$, has a length of 5, and $\tan(\theta) = 5$, use the definitions of sine and tangent to evaluate $\sin(\tan^{-1}(5))$.

\[
\sin(\theta) = \frac{\text{opp}}{\text{hyp}} = \frac{5}{\sqrt{26}}
\]

\[
\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{5}{\sqrt{26}}
\]

\[
\cos^2(\theta) + \sin^2(\theta) = 1
\]

\[
c = \sqrt{1 + 25} = \sqrt{26}
\]

17. Given two functions, $f(x) = \frac{3}{x+1}$ and $g(x) = \frac{x}{x+1}$, which function represents $h(x) = f(g(x))$?

\[
a. \quad h(x) = \frac{3}{x+1}
\]

\[
b. \quad h(x) = \frac{3x}{(x+1)^2}
\]

\[
c. \quad h(x) = \frac{3x+x+1}{x+1} = \frac{3x+3}{2x+1}
\]

\[
d. \quad h(x) = \frac{x}{x+1}
\]

\[e. \quad h(x) = \frac{2x+3}{2x+1}
\]

18. Evaluate the exponential function, $f(x) = 4 - 3^{-x}$, at $x = 2$.

\[
f(x) = 4 - 3^{-2} = 4 - \frac{1}{3^2} = 4 - \frac{1}{9} = \frac{36}{9} - \frac{1}{9} = \frac{35}{9}
\]

\[d. \quad 35/9
\]
19. Find the inverse of the one-to-one function \( f(x) = (x + 3)^3 \).

a. \( f^{-1}(x) = \sqrt[3]{x - 3} \)

b. \( f^{-1}(x) = \sqrt[3]{x + 3} \)

c. \( f^{-1}(x) = \frac{\sqrt[3]{x} - 3}{3} \)

d. \( f^{-1}(x) = \frac{\sqrt[3]{x} - 3}{3} \)

e. \( f^{-1}(x) = \frac{\sqrt[3]{x} - 3}{3} \)

20. Which of the following is the simplification of \((4 + i)(3 - i)(1 + i - i^2)\)?

a. \( 27 + 11i \)

b. \( 9 + i \)

c. \( 24 + i \)

d. \( 24 - i^2 \)

e. \( 9 + 3i \)
21. Write the equation, in slope intercept form, for the line through points (2,1) and (4,5).

\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{4 - 2} = 1 \]

\[ y - y_1 = m(x - x_1) \]

\[ y - 1 = 1(x - 2) \]

\[ y = x - 1 \]

22. Convert from radians to degrees: \( \frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = \frac{5 \cdot 180^\circ}{3} = 300^\circ \)

23. Add \( \frac{1}{4} + \frac{2}{7} \)

\[ \frac{1}{4} \left( \frac{7}{7} \right) + \frac{2}{7} \left( \frac{4}{4} \right) = \frac{7}{28} + \frac{8}{28} = \frac{15}{28} \]

24. Simplify: \( \frac{a^6 b^7 b^{-4}}{a^3} = a^{6-3} b^{7-4} = a^3 b^3 \)

25. Factor: \( x^2 + 9x + 20 \)

\( (x + 4)(x + 5) \)

26. Divide and simplify: \( \frac{4}{5} + \frac{8}{3} \)

\[ \frac{4}{5} \cdot \frac{3}{3} = \frac{3}{5} \]

27. Find exactly: \( \tan(30^\circ) = \frac{\sin(30^\circ)}{\cos(30^\circ)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2} \cdot \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \)

28. Solve for \( x \) where: \( 8x = 2x + 24 \)

\[ 6x = 24 \]

\[ x = 4 \]

29. Find the distance between the points given by \((-2,4)\) and \((3,3)\).

\[ \sqrt{(3 - (-2))^2 + (3 - 4)^2} = \sqrt{(3 + 2)^2 + (3 - 4)^2} = \sqrt{5^2 + (-1)^2} = \sqrt{26} \]

30. Sketch the graph of the function given by \( f(x) = -x^2 + 1 \). Include axes with tick marks.
31. Combine the logs below and use the rules for expanding or collapsing logs to solve for \( x \)

\[
\log_2(x) + \log_2(x + 7) - \log_2(x^2) = 3
\]

\[
\log_2\left(\frac{x(x + 7)}{x^2}\right) = 3
\]

\[
\log_2\left(\frac{x^2 + 7x}{x^2}\right) = 2
\]

\[
x^2 \cdot \left(\frac{x^2 + 7x}{x^2}\right) = 8x^2
\]

\[
x^2 + 7x = 8x^2
\]

\[
-7x^2 - 7x = 0
\]

\[
7x(x - 1) = 0
\]

\[
x = 0 \quad x = 1
\]

but \( \log_2(0) \) is undefined

\[
x = 1 \quad \text{only}
\]

32. A regular hexagon can be constructed from six equilateral triangles. The area of a triangle is \( A = \frac{1}{2}bh \)

If each side has a length of 2, calculate the area of the hexagon.

\[
data:image/png;base64,iVBORw0KGgoAAAANSUhEUgAAAK8AAAFqCAIAAADs9X+hAAAAGXRFWHRTb2Z0d2FyZQBBZG9iZSBJbWFnZVJlYWR5ccllPAAAA4XRFWHRDIBBAAgEAAACQCAAAAAfif5/bAAAAmJLR0pRiKp8AAAA0RSTlDAAAAAAAAAAAJISEAAAAAAAAAAAAAClI8AAAAAAzklEQAAAAA1JREFUeNpiGQkQ4tXyAAAAASUVORK5CYII=
\]
33. Sketch a graph of the function, \( y = -(x - 1)^3(x + 2)(x^2 - 4) \).

Points will be given for correct general shape, end behavior, and correct axis intercepts.

(Use of “test points” is not required, correct scale is not required)
34. Given the function \( f(x) = \frac{x^2 + 5x + 6}{x^2 - 4} \)

a. Simplify the fraction by factoring

\[
\frac{(x + 3)(x - 2)}{(x - 2)(x + 3)} = \frac{x + 3}{x - 2}
\]

b. Simplify the fraction by long division.

\[
\frac{1}{x^2 - 4} \quad \frac{x^2 + 5x + 6}{x^2 - 4} = 1 + \frac{5x + 10}{x^2 - 4}
\]

\[
= 1 + \frac{5(x + 2)}{(x - 2)(x + 2)}
\]

\[
= 1 + \frac{5}{x - 2}
\]

c. Does this graph have asymptotes, holes, both, or neither? If asymptotes or holes exist, list them below.

(Sketch is not required, but may help you)

Horizontal: \( y = 1 \)

Vertical: \( x = 2 \)

Hole at: \( x = -2 \) \( \Rightarrow (-2, -\frac{1}{4}) \)

\[
-2 + 3 = \frac{1}{-2 - 2} = -4
\]
35. Given the following parabola,

\[ y = f(x) = x^2 - 4x + 5 \]

a. Does this function intercept the x-axis? If so, give coordinates of any intercepts. If not, write "NO X-INTERCEPT"

The discriminant is given by \( b^2 - 4ac = (-4)^2 - 4(1)(5) = 16 - 20 = -4 \) no real roots.

no real x-intercepts

b. Does this function intercept the y-axis? If so, give coordinates of any intercepts. If not, write "NO Y-INTERCEPT"

\[ f(0) = 0^2 - 4(0) + 5 = 5 \]

\[ (0, 5) \]

c. Find the vertex of the parabola. (Hint: complete the square)

\[ f(x) = (x^2 - 4x) + 5 \]

\[ (-2)^2 + f(-2) = (x^2 - 4x + (-2)^2) + 5 \]

\[ 4 + f(-2) = (x - 2)^2 + 5 \]

\[ f(-2) = (x - 2)^2 + 1 \]

\[ f(x) = (x - 2)^2 + 1 \]

36. The two lines below intersect at the point (1,3). Since that point is on both lines, solve for \( a \) and \( b \)

\[ ax + by = 3 \]

\[ ax - by = 8 \]

\[ x = 1 \quad y = 3 \]

\[ a(1) + b(3) = 3 \]

\[ a + 3b = 3 \]

\[ a(1) - b(3) = 8 \]

\[ a - 3b = 8 \]

\[ a + 3b = 3 \]

\[ + \quad a - 3b = 8 \]

\[ 2a = 11 \]

\[ a = \frac{11}{2} \]

\[ 3b = 3 - a \]

\[ b = \frac{3 - a}{3} = 1 - \frac{1}{3}a \]

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\[ = 1 - \frac{1}{3} \left( \frac{11}{2} \right) = \frac{6}{6} - \frac{11}{6} = -\frac{5}{6} \]
37. Given the function \( y = f(x) = \cos(2x) + 3 \)

   a. What are the domain and range of \( f(x) \)
      
      \[
      \text{domain: } (-\infty, \infty) \quad \text{range: } (2, 4)
      \]

   b. What is the period of \( f(x) \)?
      
      \[
      \frac{1}{2} (2\pi) = \pi
      \]

   c. Sketch at least TWO periods of the graph of \( f(x) \)
38. For the trig function, \( y = \tan(x) \), Sketch at least TWO periods of the graph of the function.

39. Simplify or evaluate the following
   a. \( \tan^{-1}(1) = \frac{\pi}{4} \)
   b. \( \log_{361}(1) = 0 \)
   c. \( \sin^{-1}(1) = \frac{\pi}{2} \)
   d. \( (3.5)^{-1} = \frac{1}{3.5} = \frac{1}{\frac{7}{2}} = \frac{2}{7} \)
   e. \( 1^{\frac{3}{7}} = 1 \)
   f. \( \sqrt{-54} = \sqrt{-1 \cdot 54} = i \cdot 3 \cdot \sqrt{6} = 3i \sqrt{6} \)