NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

Instructions:

1. You have three hours to complete this test.
2. Fill out the Scantron form
   a. Bubble in your Alpha Number
   b. Bubble in Version 0 on the top line of the Version section
   c. Write your name and section in the top right of the form
3. Read each problem carefully. Ask questions if you do not understand the problem.
4. Read through the exam first. Start with the easy problems. You may complete the exam in any order.
5. Record all Multiple Choice answers on the Scantron form.
6. **SHOW ALL YOUR WORK** – partial credit will be awarded for correct work.
7. **Calculators are NOT ALLOWED for use on any part of this test**

HONOR COMMITMENT

By signing below, I certify that:

1. the work that I will write in this exam represents my own work and my best understanding of the material; and
2. I will neither give assistance to another student nor receive assistance during this exam except that which comes from my instructor.

_____

(signature)

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet

1. Which of the following points is on the circle $(x-3)^2+(y+1)^2=25$

   a. $(3,-1)$
   b. $(-1,2)$
   c. $(-2,0)$
   d. $(-2,4)$
   e. $(-3,1)$

$(x-h)^2+(y-k)^2=25$

---

2. Which number is closest to the value of $x=\log_3(20)$?

   a. 0.15
   b. 6.7
   c. 59
   d. 2.7
   e. 7999

$\log_3(9)=\log_3(3^2)=2$
$\log_3(27)=\log_3(3^3)=3$

---

3. Which line represents the inverse function of $y=3x-7$?

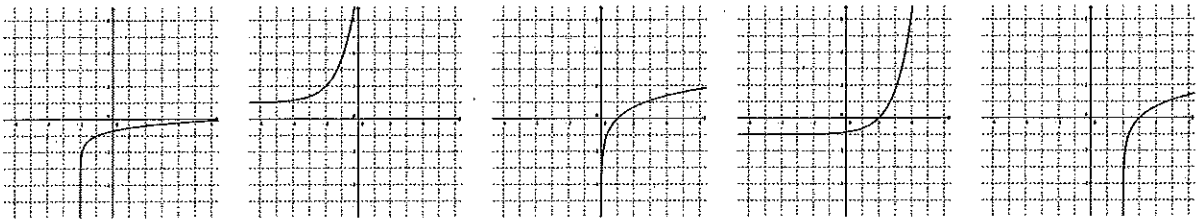
   a. $y=-\frac{1}{3}x+7$
   b. $y=-3x+7$
   c. $y=\frac{1}{3}x-\frac{1}{7}$
   d. $y=-\frac{1}{3}x+\frac{7}{3}$
   e. $y=\frac{1}{3}x+\frac{7}{3}$

$3x=y+7$
$x=\frac{y+7}{3}=\frac{1}{3}y+\frac{7}{3}$
$y=\frac{1}{3}x+\frac{7}{3}$
$f^{-1}(x)=\frac{1}{3}x+\frac{7}{3}$

---

4. Which graph represents $y=\log(x+2)-1$

horizontal shift left 2
vertical shift down 1



a. b. c. d. e.

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet

5. If $3x + 5 = 7$, which of the following is true

   a. $9x^2 + 25 = 49$
   b. $3\ln(x) = \ln(2)$
   c. $1000^x = 100$
   d. $18x + 5 = 42$
   e. None of the above

$3x = 2$

$x = \frac{2}{3}$

$(1000)^{\frac{2}{3}} = (10^3)^{\frac{2}{3}}$

$= 10^{3 \cdot \frac{2}{3}} = 10^2 = 100$

---

6. Which of the following is equal to $\sqrt{2(9+36)}$

   a. $3\sqrt{10}$
   b. $9\sqrt{2}$
   c. $\sqrt{18} + \sqrt{72}$
   d. 45
   e. $6\sqrt{5}$

$\sqrt{2(45)}$

$\sqrt{2 \cdot 3 \cdot 3 \cdot 5}$

$3\sqrt{10}$

---

7. Which is the domain of the following function, $f(x) = \frac{\ln(9-x)}{9+x}$

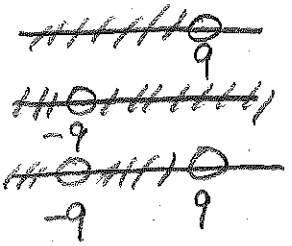
   a. $(-\infty, \infty)$
   b. $(-\infty, -9) \cup [9, \infty)$
   c. $(-\infty, -9) \cup (-9, 9) \cup (9, \infty)$
   d. $(-\infty, -9] \cup [9, \infty)$
   e. $(-\infty, -9) \cup (-9, 9)$

$9 - x > 0$

$9 > x$

$9 + x \neq 0$

$x \neq -9$



---

8. Which of the following is a line through the point $(-7,2)$ and parallel to $-7x + 3y = 61$?

   a. $y = \frac{3}{7}x - \frac{2}{7}$
   b. $y = \frac{7}{3}x + \frac{61}{3}$
   c. $y = \frac{7}{3}x + \frac{55}{3}$
   d. $y = -\frac{7}{3}x - \frac{55}{3}$
   e. None of the above

$3y = 7x + 61$

$y = \frac{7}{3}x + \frac{61}{3}$

$m = \frac{7}{3}$

$y - 2 = \frac{7}{3}(x - (-7))$

$y - 2 = \frac{7}{3}(x + 7)$

$y - 2 = \frac{7}{3}x + \frac{49}{3}$

$y = \frac{7}{3}x + \frac{49}{3} + 2$

$y = \frac{7}{3}x + \frac{49}{3} + \frac{6}{3}$

$y = \frac{7}{3}x + \frac{55}{3}$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet

9. Solve the equation $(5x-5)^{1/3}+10=13$

a. $\frac{5}{32}$
b. $\frac{32}{5}$
c. $\frac{27}{5}$
d. $-\frac{27}{5}$
e. none of the above

$(5x-5)^{\frac{1}{3}} = 3$

$\left((5x-5)^{\frac{1}{3}}\right)^3 = 3^3$

$5x-5 = 27$

$5x = 32$

$x = \frac{32}{5}$

10. Solve the equation, $3^{(6-3x)}=\frac{1}{27}$

a. 3
b. -3
c. 9
d. 1/9
e. 0

$3^{(6-3x)} = \frac{1}{3^3}$

$3^{(6-3x)} = 3^{-3}$

$\log_3(3^{(6-3x)}) = \log_3(3^{-3})$

$6-3x = -3$

$9 = 3x$

$x = 3$

11. Given the following equation, $10^{0.9542}=9$, which equation below is true?

a. $9 = \log(0.9542)$
b. $0.9542 = \log_9(10)$
c. $10 = \log_9(0.9542)$
d. $0.9542 = \log(9)$
e. None of the above

$\log_{10}(9) = 0.9542$

12. Find the value of $\log_{12}(6)+\log_{12}(24)$

a. $\frac{5}{2}$
b. 2
c. $\frac{1}{2}$
d. $\frac{1}{3}$
e. 3

$\log_{12}(6\cdot 24)$

$\log_{12}(144)$

$\log_{12}(12^2)$

2

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet – Calculator use is not allowed

13. Write the following expression in condensed form

$$-3[4\ln(x-4) - \ln(x+4)]$$

a. $-12\ln\left(\frac{(x-4)^4}{(x+4)}\right)$

b. $\ln\left(\frac{(x+4)^3}{(x-4)^{12}}\right)$

c. $\ln\left(\frac{(x+4)^4}{(x-4)^3}\right)$

d. $\ln\left(\sqrt[3]{\frac{(x+4)^4}{(x-4)^4}}\right)$

e. $\ln\left(\frac{3}{(x-4)^4(x+4)}\right)$

$-3\left[\ln(x-4)^4 - \ln(x+4)\right]$

$-3\left[\ln\left(\frac{(x-4)^4}{x+4}\right)\right]$

$\ln\left[\left(\frac{(x-4)^4}{x+4}\right)^{-3}\right]$

$\ln\left[\left(\frac{x+4}{(x-4)^4}\right)^3\right]$

---

14. What is the y-intercept of the following piecewise function?

$$y = f(x) = \begin{cases} x^2+1, & x < 1 \\ -(x+3), & x \geq 1 \end{cases}$$

a. $(0,-3)$
b. $(-3,0)$
c. $(0,1)$
d. $(-1,0)$
e. There is no y-intercept

$f(0) = 0^2 + 1 = 1$

$(0,1)$

---

15. Find the angle below that is co-terminal with $\theta = -\frac{61\pi}{6}$ (i.e. points in the same direction)

a. $\frac{\pi}{6}$
b. $\frac{49\pi}{6}$
c. $\frac{5\pi}{6}$
d. $-\frac{\pi}{6}$
e. None of the above

$-\frac{61}{6}\pi + 2\pi = -\frac{61}{6}\pi + \frac{12}{6}\pi = -\frac{49}{6}\pi$

$-\frac{49}{6}\pi + 2\pi = -\frac{49}{6}\pi + \frac{12}{6}\pi = -\frac{37}{6}\pi$

$-\frac{37}{6}\pi + 2\pi = -\frac{37}{6}\pi + \frac{12}{6}\pi = -\frac{25}{6}\pi$

and so on $-\frac{13}{6}\pi, -\frac{1}{6}\pi$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet – Calculator use is not allowed

16. If the side opposite angle, θ, has a length of 5, and $\tan(\theta) = 5$, use the definitions of sine and tangent to evaluate $\sin(\tan^{-1}(5))$.

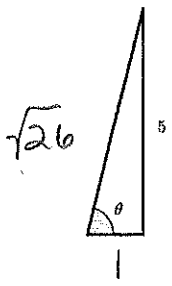
a. $\frac{5}{26}$
(b.) $\frac{5\sqrt{26}}{26}$
c. $\frac{\sqrt{26}}{5}$
d. $\frac{\sqrt{26}}{26}$
e. $\frac{25}{26}$

$\tan(\theta) = \frac{opp}{adj}$

$adj = \frac{opp}{\tan(\theta)} = \frac{5}{5} = 1$

$1^2 + 5^2 = c^2$

$c = \sqrt{1+25} = \sqrt{26}$



$\sin(\theta) = \frac{opp}{hyp} = \frac{5}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$

17. Given two functions, $f(x) = \frac{3}{x+1}$ and $g(x) = \frac{x}{x+1}$, which function represents $h(x) = f(g(x))$

a. $h(x) = \frac{3}{x}$
b. $h(x) = \frac{3x}{(x+1)^2}$
c. $h(x) = \frac{3+x}{x+1}$
d. $h(x) = \frac{3}{x+4}$
(e.) $h(x) = \frac{3x+3}{2x+1}$

$\frac{3}{g+1} = \frac{3}{\frac{x}{x+1}+1} = \frac{3}{\frac{x}{x+1}+\frac{x+1}{x+1}} = \frac{3}{\frac{2x+1}{x+1}} = \frac{\frac{3}{1}}{\frac{2x+1}{x+1}}$

$= \frac{3}{1} \cdot \frac{x+1}{2x+1} = \frac{3x+3}{2x+1}$

18. Evaluate the exponential function, $f(x) = 4 - 3^{-x}$, at $x = 2$.

a. 37/9
b. 1/3
c. 10/3
(d.) 35/9
e. 13

$f(2) = 4 - 3^{-2} = 4 - \frac{1}{3^2} = 4 - \frac{1}{9}$

$= \frac{36}{9} - \frac{1}{9} = \frac{35}{9}$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART ONE: MULTIPLE CHOICE (50%)
Mark all of your answers on the Scantron Sheet – Calculator use is not allowed

19. Find the inverse of the one-to-one function $f(x) = (x+3)^3$.

a. $f^{-1}(x) = \sqrt[3]{x-3}$
b. $f^{-1}(x) = \sqrt[3]{x}+3$
c. $f^{-1}(x) = \sqrt[3]{x}-3$
d. $f^{-1}(x) = \sqrt[3]{x}-27$
e. $f^{-1}(x) = \sqrt[3]{x-27}$

$y = (x+3)^3$
$\sqrt[3]{y} = x+3$
$x = \sqrt[3]{y} - 3$
$y = \sqrt[3]{x} - 3$
$f^{-1}(x) = \sqrt[3]{x} - 3$

20. Which of the following is the simplification of $(4+i)(3-i)(1+i-i^2)$

a. $27+11i$
b. $9+i$
c. $24+i$
d. $24-i^3$
e. $9+3i$

$(12 - 4i + 3i - i^2)(1 + i - (-1))$
$(12 - i - (-1))(1 + i + 1)$
$(12 - i + 1)(2 + i)$
$(13 - i)(2 + i)$
$26 + 13i - 2i - i^2$
$26 + 11i - (-1)$
$26 + 11i + 1$
$27 + 11i$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART II: SHORT ANSWER (10%)
Write your answers in the space provided – Calculator use is not allowed

21. Write the equation, in slope intercept form, for the line through points (2,1) and (4,5)

$m = \frac{5-1}{4-2} = \frac{4}{2} = 2$ $\quad y-1 = 2(x-2)$ $\quad y = 2x-3$
$y-1 = 2x-4$

22. Convert from radians to degrees: $\frac{5\pi}{3} \cdot \frac{360^\circ}{2\pi} = \frac{5 \cdot 180^\circ}{3} = 300^\circ$

23. Add $\frac{1}{4}+\frac{2}{7}$ $\quad \frac{1}{4}\frac{(7)}{(7)} + \frac{2}{7}\frac{(4)}{(4)} = \frac{7}{4\cdot 7} + \frac{8}{4\cdot 7} = \frac{15}{4\cdot 7} = \frac{15}{28}$

24. Simplify: $a^6b^7b^{-4}/a^3$ $\quad \frac{a^6 b^7 b^{-4}}{a^3} = a^{6-3} b^{7-4} = a^3 b^3$

25. Factor: $x^2 + 9x + 20$ $\quad (x+4)(x+5)$

26. Divide and simplify: $\frac{4}{5} \div \frac{8}{3}$ $\quad \frac{4}{5} \cdot \frac{3}{8_2} = \frac{3}{10}$

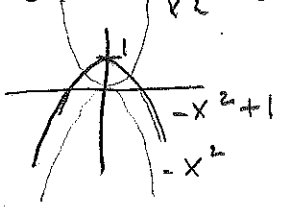
27. Find exactly: tan(30°) $\quad = \frac{\sin(30^\circ)}{\cos(30^\circ)} = \frac{\sin(\frac{1}{6}\pi)}{\cos(\frac{1}{6}\pi)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

28. Solve for x where: $8x = 2x + 24$ $\quad 6x = 24$
$x = 4$

29. Find the distance between the points given by (−2,4) and (3,3)

$\sqrt{(3-(-2))^2+(3-4)^2} = \sqrt{(3+2)^2+(3-4)^2} = \sqrt{5^2+(-1)^2} = \sqrt{26}$

30. Sketch the graph of the function given by $f(x) = -x^2 + 1$. Include axes with tick marks.



NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

31. Combine the logs below and use the rules for expanding or collapsing logs to solve for x

$$\log_2(x) + \log_2(x+7) - \log_2(x^2) = 3$$

$\log_2\left(\frac{x(x+7)}{x^2}\right) = 3$

$\log_2\left(\frac{x^2+7x}{x^2}\right) = 2^3$

$x^2 \cdot \left(\frac{x^2+7x}{x^2}\right) = (8) \cdot x^2$

$x^2 + 7x = 8x^2$

$7x^2 - 7x = 0$

$7x(x-1) = 0$

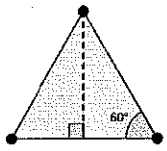
$7x = 0$ $\quad x-1=0$

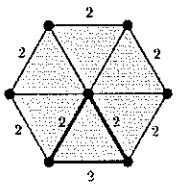
$x = 0$ $\quad x = 1$

but $\log_2(0)$ is undefined

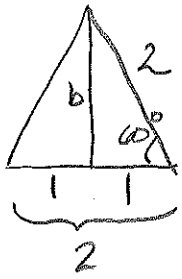
$x = 1$ only

32. A regular hexagon can be constructed from six equilateral triangles. The area of a triangle is $A = \frac{1}{2}bh$





If each side has a length of 2, calculate the area of the hexagon.



$1^2 + b^2 = 2^2$

$1 + b^2 = 4$

$b^2 = 3$

$b = \sqrt{3}$

OR $\sin(60°) = \frac{\text{opp}}{\text{hyp}}$

$\sin\left(\frac{1}{3}\pi\right) = \frac{b}{2}$

$b = 2\sin\left(\frac{1}{3}\pi\right)$

$b = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

$A = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3}$

$6A = 6\sqrt{3}$

NAME: _____ ALPHA: _____

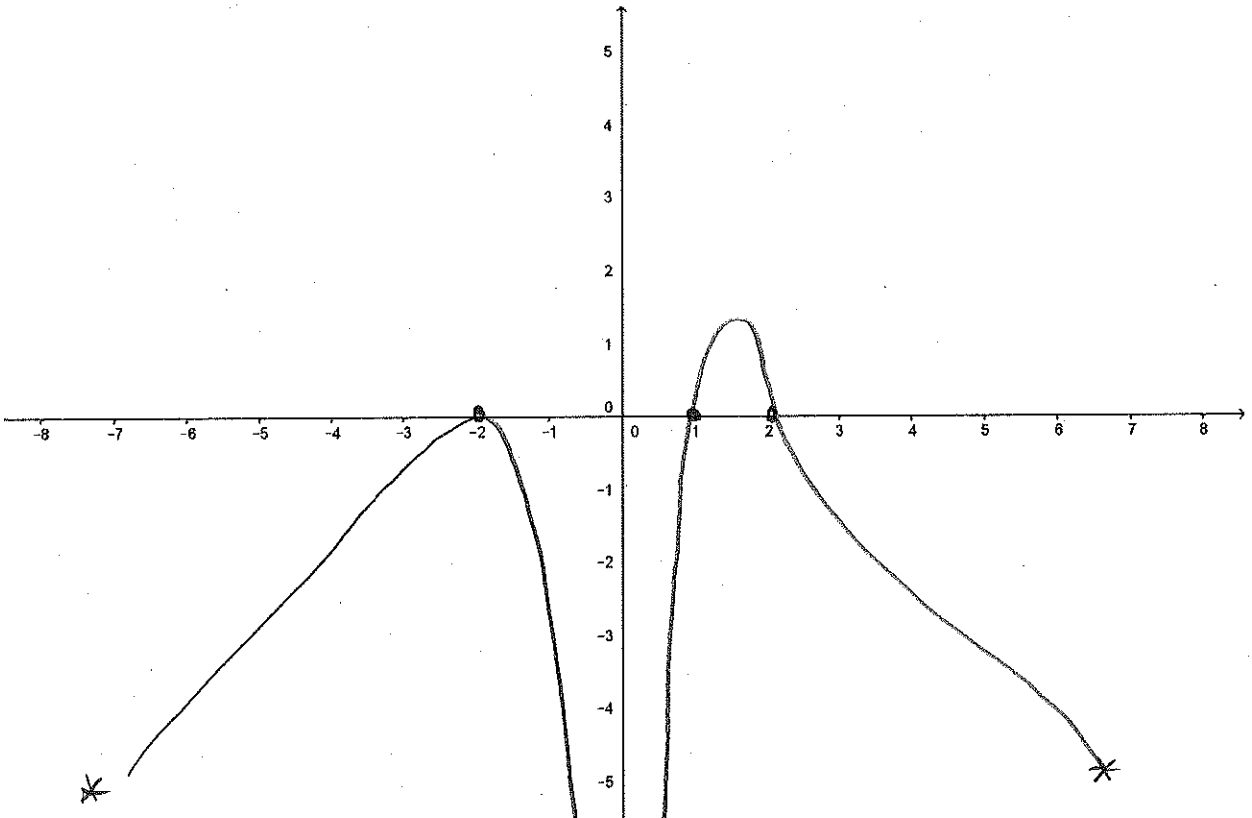
PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

33. Sketch a graph of the function, $y = -(x-1)^3(x+2)(x^2-4)$.

Points will be given for correct general shape, end behavior, and correct axis intercepts.

(Use of "test points" is not required, correct scale is not required)



$3+1+2=6$

$-x^{even}$: $+$ $\quad$ -8

$f(0) = -(0-1)^3(0+2)(0^2-4)$

$= -(-1)^3(2)(-4)$

$= -(-1)(2)(-4)$

$= -8$

$x-1=0$ $\quad$ $x+2=0$ $\quad$ $x^2-4=(x+2)(x-2)=0$

$x=1$ $\quad$ $x=-2$ $\quad$ $x=-2$ $\quad$ $x=2$

multiplicity 1 through $\quad$ multiplicity 2 bounce $\quad$ multiplicity 1 through

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

34. Given the function $f(x) = \frac{x^2+5x+6}{x^2-4}$

a. Simplify the fraction by factoring

$$\frac{(x+3)(x+2)}{(x-2)(x+2)} = \frac{x+3}{x-2}$$

b. Simplify the fraction by long division.

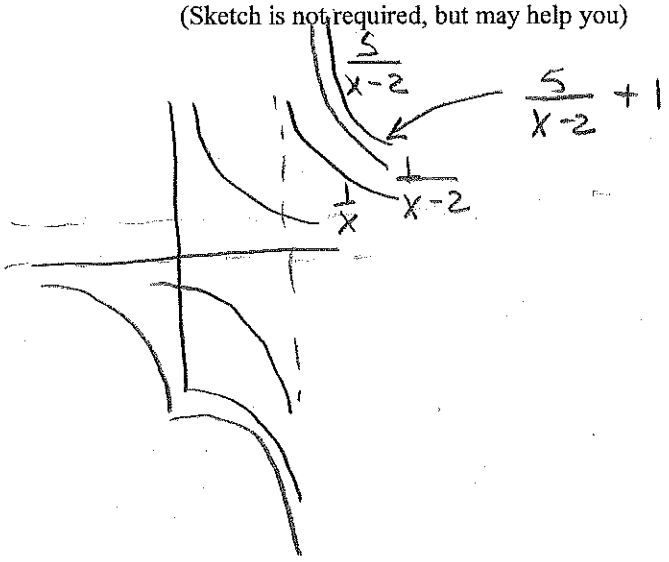
$$\begin{array}{r} 1 \\ x^2-4 \overline{)\, x^2+5x+6} \\ -x^2 \quad +4 \\ \hline 5x+10 \end{array}$$

$$= 1 + \frac{5x+10}{x^2-4}$$

$$= 1 + \frac{5(x+2)}{(x-2)(x+2)}$$

$$= 1 + \frac{5}{x-2}$$

c. Does this graph have asymptotes, holes, both, or neither? If asymptotes or holes exist, list them below.
(Sketch is not required, but may help you)



horizontal $y = 1$

vertical $x = 2$

hole at $x = -2 \Rightarrow \left(-2, -\frac{1}{4}\right)$

$$\frac{-2+3}{-2-2} = \frac{1}{-4}$$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

35. Given the following parabola,

$$y = f(x) = x^2 - 4x + 5$$

a. Does this function intercept the x-axis? If so, give coordinates of any intercepts. If not, write "NO X-INTERCEPT"

discriminant $= b^2 - 4ac = (-4)^2 - 4(1)(5)$
$= 16 - 20 = -4$ no real roots.
no real x intercepts

b. Does this function intercept the y-axis? If so, give coordinates of any intercepts. If not, write "NO Y-INTERCEPT"

$f(0) = 0^2 - 4(0) + 5 = 0 - 0 + 5 = 5$
$(0, 5)$

c. Find the vertex of the parabola. (Hint: complete the square)

$f(x) = (x^2 - 4x) + 5$
$(-2)^2 + f(x) = (x^2 - 4x + (-2)^2) + 5$
$4 + f(x) = (x-2)^2 + 5$
$f(x) = (x-2)^2 + 1$

$(h, k) = (2, 1)$

---

36. The two lines below intersect at the point (1,3). Since that point is on both lines, solve for a and b

$$ax + by = 3$$
$$ax - by = 8$$

$x = 1 \quad y = 3$

$a(1) + b(3) = 3$
$a + 3b = 3$

$a(1) - b(3) = 8$
$a - 3b = 8$

$a + 3b = 3$
$+ \; a - 3b = 8$
$2a = 11$

$a = \frac{11}{2}$

$3b = 3 - a$

$b = \frac{3-a}{3} = 1 - \frac{1}{3}a$

$= 1 - \frac{1}{3}\left(\frac{11}{2}\right) = \frac{6}{6} - \frac{11}{6} = -\frac{5}{6}$

NAME: _____ ALPHA: _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

37. Given the function $y = f(x) = \cos(2x) + 3$

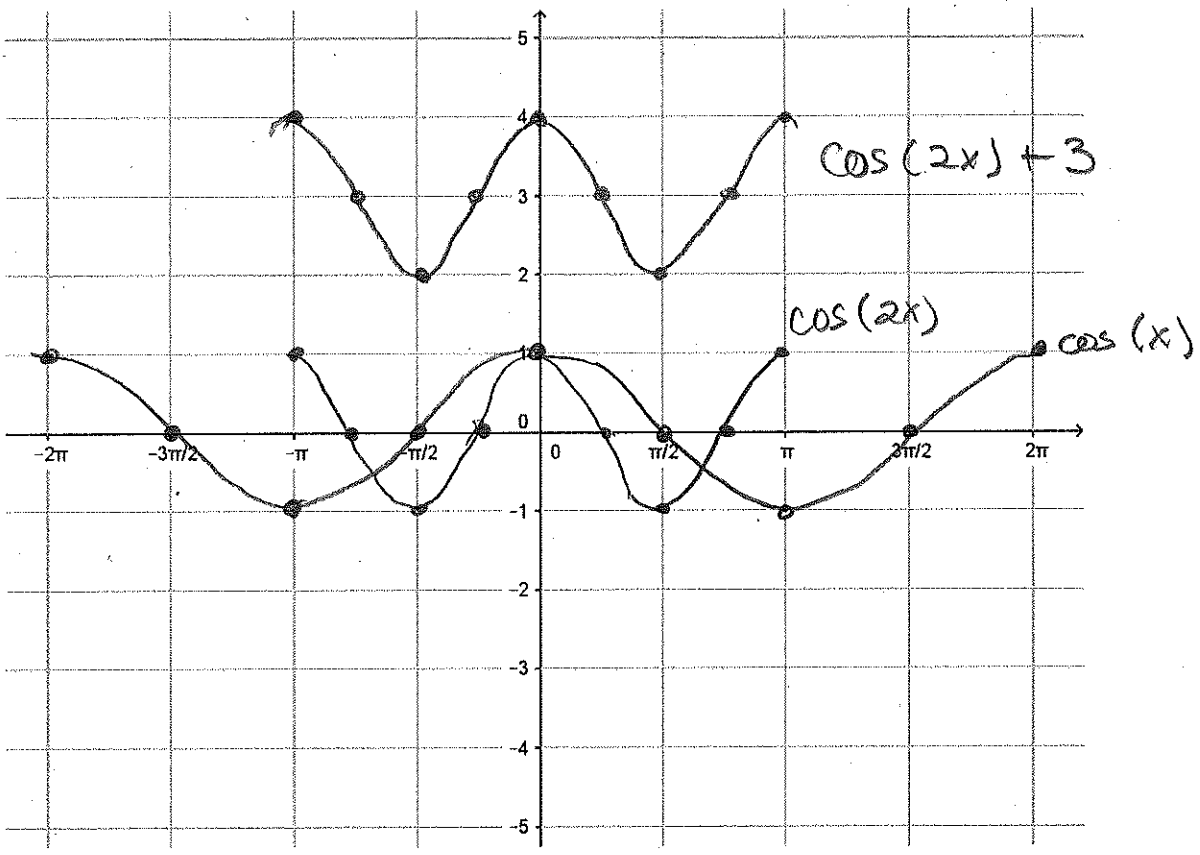
a. What are the domain and range of $f(x)$

domain: $(-\infty, \infty)$ range: $(2, 4)$

b. What is the period of $f(x)$?

$\frac{1}{2}(2\pi) = \pi$

c. Sketch at least TWO periods of the graph of $f(x)$

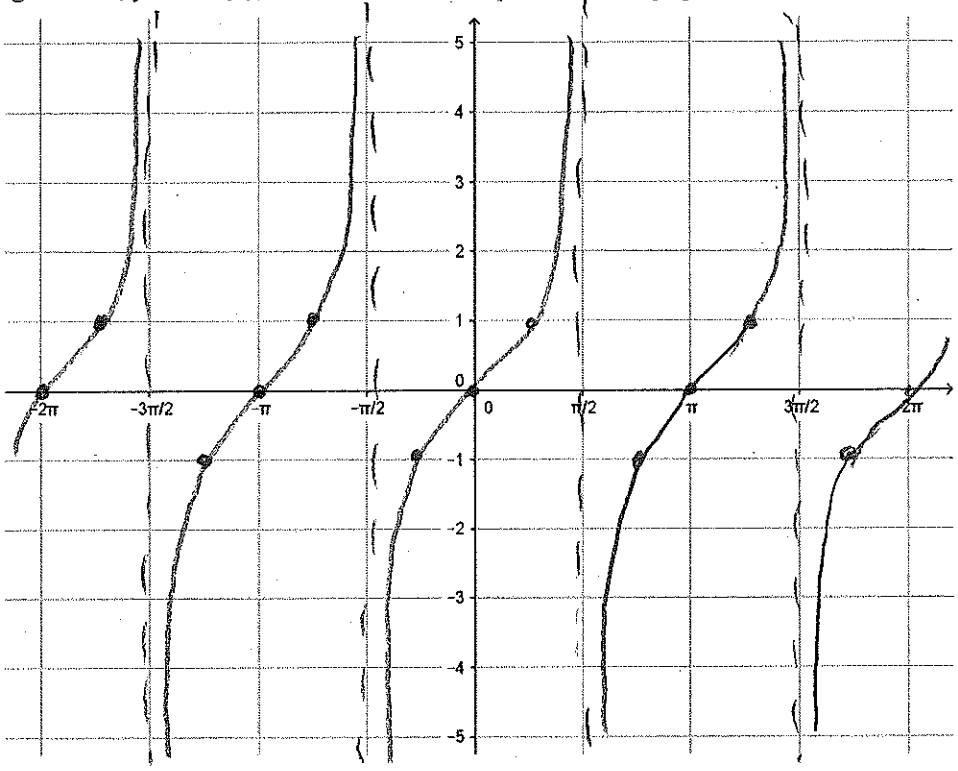


NAME: _____ **ALPHA:** _____

PRECALCULUS (SM005)
FINAL EXAM – FALL 2015
15 DEC 2015

PART III: FREE RESPONSE (40%)
Write your answers in the space provided – Calculator use is not allowed

38. For the trig function, $y = \tan(x)$, Sketch at least TWO periods of the graph of the function



39. Simplify or evaluate the following

a. $\tan^{-1}(1)$ $\tan(\theta) = 1$ $\frac{\sin(\theta)}{\cos(\theta)} = 1$ $\sin(\theta) = \cos(\theta)$ $\tan^{-1}(1) = \frac{1}{4}\pi$

b. $\log_{361}(1)$ $= 0$.

c. $\sin^{-1}(1)$ $\frac{1}{2}\pi$

d. $(3.5)^{-1}$ $\frac{1}{3.5} = \frac{1}{3+\frac{5}{10}} = \frac{1}{\frac{30}{10}+\frac{5}{10}} = \frac{1}{\frac{35}{10}} = \frac{1}{\frac{35}{10}} = \frac{1}{1}\cdot\frac{10}{35} = \frac{10}{35} = \frac{2}{7}$

e. $1^{5/7}$ $= 1$

f. $\sqrt{-54}$ $= \sqrt{-1}\sqrt{54} = \sqrt{-1}\sqrt{2\cdot 3\cdot 3\cdot 3} = i\cdot 3\sqrt{6} = 3\sqrt{6}\,i$