SOLUTIONS KEY

Name: _____ Alpha: _____

Precalculus SM005 Fall 2017-2018

Final Exam

December 12, 2017

Course coordinator: Linda Shivok <lshivok@usna.edu>

General Instructions:

- You will have 3 hours to complete this exam. The exam contains a total of 220 points.
- Immediately fill out the top of the Scantron form using a number 2 pencil:
  - Bubble in your alpha.
  - Do not bubble in a version code.
  - Write your name and section.
- Read through the entire exam first. Start with the easy problems. You may complete the exam in any order.
- Ask questions if you do not understand the problem.
- No calculators, notes, or other resources are allowed. You must work alone.

Multiple Choice:

- There are 30 multiple choice questions, Questions 1 through 30.
- Each question is worth 4 points. The exam contains a total of 120 multiple choice points.
- You are required to complete all 30 questions.
- Record all multiple choice answers on the Scantron form.
- There is no partial credit for these questions. You will not be given any credit for work or answers in the test booklet.

Free Response:

- There are 17 free response questions, Questions 31 through 47.
- Questions 31 through 33 are worth 10 points each. Questions 34 through 47 are worth 5 points each. The exam contains a total of 100 free response points.
- You are required to complete all 17 questions.
- Show all your work in the test booklet.
- Partial credit may be awarded for these questions.

1. (4 points) In which quadrant(s) is $sec(x)$ negative?

$$sec(x)=\frac{1}{\cos(x)}$$

$\cos(x)$ is negative in quadrants II and III, so $sec(x)$ is negative in quadrants II and III.

A. III and IV — $\sin(x)$ and $csc(x)$ negative

B. II and III

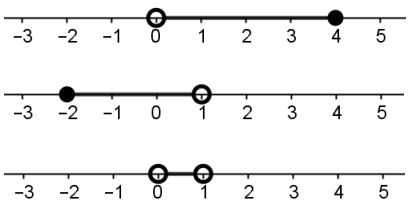
C. I and III — incorrect sign, incorrect function

D. I and IV — incorrect sign

E. II only — quadrant III omitted

2. (4 points) Determine $(0,4]\cap[-2,1)$. Give your answer in interval notation.

Intersection is what two sets have in common.



A. $[0,1]$ — incorrect treatment of endpoints

B. $[-2,0)\cup(0,1)\cup(1,4]$ — union, incorrect holes

C. $(0,1)$

D. $(-\infty,0)\cup(0,1)\cup(1,\infty)$ — union, incorrect treatment of extremes

E. $[-2,4]$ — union

3. (4 points) Determine the values of sine and cosine of $\frac{5}{6}\pi$.

From the unit circle

A. $\sin\left(\frac{5}{6}\pi\right)=\frac{1}{2},\cos\left(\frac{5}{6}\pi\right)=\frac{\sqrt{3}}{2}$ — incorrect quadrant

B. $\sin\left(\frac{5}{6}\pi\right)=\frac{1}{2},\cos\left(\frac{5}{6}\pi\right)=\frac{-\sqrt{3}}{2}$

C. $\sin\left(\frac{5}{6}\pi\right)=\frac{\sqrt{3}}{2},\cos\left(\frac{5}{6}\pi\right)=\frac{-1}{2}$ — reversed values

D. $\sin\left(\frac{5}{6}\pi\right)=\frac{\sqrt{3}}{2}, \cos\left(\frac{5}{6}\pi\right)=\frac{1}{2}$ incorrect quadrant, reversed values

E. $\sin\left(\frac{5}{6}\pi\right)=\frac{-1}{2}, \cos\left(\frac{5}{6}\pi\right)=\frac{\sqrt{3}}{2}$ incorrect quadrant

4. (4 points) Let $f(x)=x^2-3x+4$. Evaluate $f(x+1)$.

$(x+1)^2-3(x+1)+4\ x^2-x+2=(x+1)(x+1)-3x-3+4$

$¿\ x^2+x+x+1-3x-3+4=x^2-x+2$

A. x^2-x+2

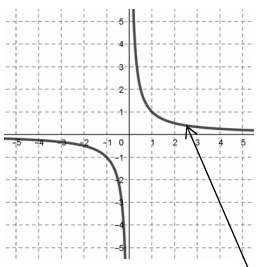
B. x^2-3x+5 — $f(x)+1$

C. x^2-x+6 — -3 not distributed to 1

D. x^2-x+8 — $-¿$ not distributed to 1

E. x^2-3x+2 — not FOILed

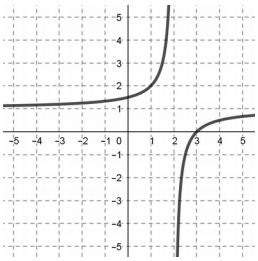
5. (4 points) Which of the following is a graph of the function $f(x)=\frac{1}{x-2}+1$?



$\frac{1}{x}$

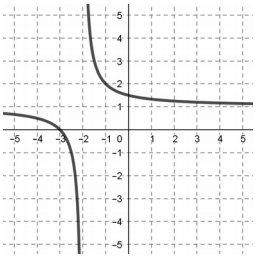
horizontal shift right 2, vertical shift up 1

A.



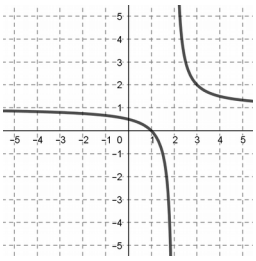
$\frac{-1}{x-2}+1$

B.

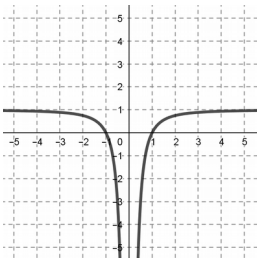


$\frac{1}{x+2}+1$

C.

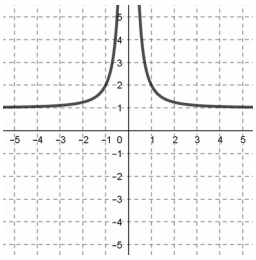


D.



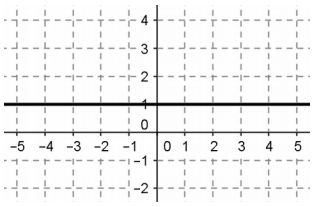
$$\frac{-1}{x^2}+1$$

E.



$$\frac{1}{x^2}+1$$

6. (4 points) What is the equation of the line shown in the graph below?



Horizontal line at $y=1$

A. $y=1$

B. $y=x$ incorrect slope, incorrect intercept

C. $y=x+1$ incorrect slope

D. $x=1$ vertical line

E. $y^2=1$ two lines

7. (4 points) The expression $\ln(e)-\ln(x^3)$ is equivalent to

$\log_e(e^1)-\ln(x^3)=1-\ln(x^3)$

A. $-\ln(x^3)$ $\ln(e)=0$

B. $\ln(e)+\ln(-x^3)$ negative distributed

C. $\dfrac{1}{\ln(x^3)}$ difference of logs is quotient of logs

D. $\dfrac{\ln(e)}{\ln(x^3)}$ difference of logs is quotient of logs

E. $1-\ln(x^3)$

8. (4 points) How many **real** solutions exist for the equation $4x^2-3x+1=0$?

Discriminant ¿ $b^2-4ac=(-3)^2-(4)(4)(1)=9-16=-7<0$, so there are no real solutions.

Therefore, the equation cannot be factored using real numbers.

A. 0, because the equation cannot be factored using whole numbers
whole numbers are not required for real solutions

B. 0, because the equation cannot be factored using real numbers

C. 1, because 1 and 4 are perfect squares
perfect squares do not guarantee real solutions

D. 2, because the degree of the polynomial is 2
a degree of 2 does not guarantee real solutions

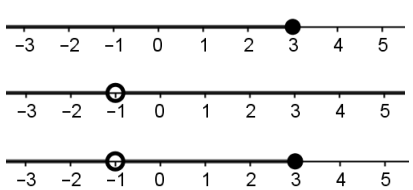
E. cannot be determined, because the equation cannot be factored using whole numbers
whole numbers are not required to determine the number of solutions

9. (4 points) Determine the domain of $f(x)=\dfrac{\sqrt{3-x}}{x+1}$. Give your answer in interval notation.

Cannot take even root of a negative number: $3-x\geq 0 \quad\rightarrow\quad 3\geq x$

Denominator cannot be 0: $x+1\neq 0 \quad\rightarrow\quad x\neq -1$

Domain is the intersection, Intersection is what two sets have in common.



A. $(-1,3]$ incorrect treatment of denominator

B. $(-1,\infty)$ incorrect treatment of denominator, radical omitted

C. $(-\infty,-1)\cup(-1,3]$

D. $(-\infty,-1]\cup[-1,3)$ incorrect treatment of endpoints

E. $(-\infty,-1)\cup(-1,\infty)$ radical omitted

10. (4 points) Determine $\cot(\pi)$.

$$\cot(\pi)=\frac{1}{\tan(\pi)}=\frac{1}{\frac{\sin(\pi)}{\cos(\pi)}}=\frac{1}{\frac{0}{-1}}=\frac{\frac{1}{1}}{\frac{0}{-1}}=\left(\frac{1}{1}\right)\left(\frac{-1}{0}\right)=¿\text{ undefined}$$

A. -1 — incorrect division by 0

B. 0 — $\tan(\pi)$

C. 1 — incorrect division by 0, incorrect sign

D. π — angle

E. undefined

11. (4 points) The expression $\log_3(81x^2)$ is equivalent to

$$\log_3(81)+\log_3(x^2)=\log_3(3^4)+2\log_3(x)=4+2\log_3(x)$$

A. $2\log_3(81x)$ — $\log_3(81x)^2$

B. $4+2\log_3(x)$

C. $4\log_3(x^2)$ — log of product is product of logs

D. $8\log_3(x)$ — log of product is product of logs

E. $4+x^2$ — log not applied to x^2

12. (4 points) Let $f(x)=\sqrt[3]{x}+2$. Determine $f^{-1}(x)$.

$y=\sqrt[3]{x}+2$

$y-2=\sqrt[3]{x}$

$(y-2)^3=x$

$(x-2)^3=y$

$f^{-1}(x)=(x-2)^3$

A. x^3+2 — inverse of $\sqrt[3]{x}$

B. $(x-2)^3$

C. $(y-2)^3$ — x and y not switched

D. x^3-2 — inverse of $\sqrt[3]{x}$

E. $(x+2)^3$ — incorrect algebra of 2

13. (4 points) Determine $\sin^{-1}\left(\frac{-1}{2}\right)$. Give your answer in radians.

$\sin(\theta)=\frac{-1}{2}$

Possible $\theta=\frac{-5}{6}\pi, -\frac{1}{6}\pi, \frac{7}{6}\pi, \frac{11}{6}\pi$

Range of inverse sine is $\left[\frac{-1}{2}\pi, \frac{1}{2}\pi\right]$

$\frac{-1}{2}\pi \le -\frac{1}{6}\pi \le \frac{1}{2}\pi$

A. $\frac{-1}{6}\pi$

B. $\frac{1}{6}\pi$ incorrect quadrant

C. $\frac{-1}{3}\pi$ incorrect angle

D. $\frac{1}{3}\pi$ incorrect angle, incorrect quadrant

E. $\frac{11}{6}\pi$ outside range

14. (4 points) Simplify $\frac{\sqrt{72}}{\sqrt{4x^6}}$ completely.

$\sqrt{\frac{72}{4x^6}}=\sqrt{\frac{18}{x^6}}=\sqrt{\frac{2\cdot 3\cdot 3}{x^6}}=\frac{3\sqrt{2}}{x^3}$

A. $\frac{3\sqrt{2}}{\sqrt{x^6}}$ $\sqrt{x^6}$ not simplified

B. $\sqrt{18}x^3$ $\sqrt{18}$ not simplified, x^3 in numerator

C. $\frac{3\sqrt{2}}{x^3}$

D. $\frac{\sqrt{72}}{2x^6}$ $\sqrt{72}$ not simplified, $\sqrt{x^6}$ simplified incorrectly

E. $3x^3\sqrt{2}$ x^3 in numerator

15. (4 points) Let $(f\circ g)(x)=\sqrt{x^2+x}$. $(f\circ g)(x)$ can be decomposed into

$g(x)=x^2+x$

$f(g)=\sqrt{g} \quad \rightarrow \quad f(x)=\sqrt{x}$

A. $f(x)=\sqrt{x^2}, g(x)=\sqrt{x}$ sum of functions

B. $f(x)=\sqrt{x}, g(x)=x^2+x$

C. $f(x)=\sqrt{x+1}, g(x)=\sqrt{x}$ product of functions

D. $f(x)=x^2+x, g(x)=\sqrt{x}$ f and g reversed

E. $f(x)=\sqrt{x^2+1}, g(x)=x$ 1 instead of x

16. (4 points) In which quadrant(s) is the angle $\frac{-25}{4}\pi$ located?

$\frac{-25}{4}\pi+2\pi=\frac{-25}{4}\pi+\frac{8}{4}\pi=\frac{-17}{4}\pi$

$\frac{-17}{4}\pi+2\pi=\frac{-17}{4}\pi+\frac{8}{4}\pi=\frac{-9}{4}\pi$

$\frac{-9}{4}\pi+2\pi=\frac{-9}{4}\pi+\frac{8}{4}\pi=\frac{-1}{4}\pi$

A. I

B. II

C. III

D. IV

E. $\frac{-25}{4}\pi$ could be located in more than one quadrant

17. (4 points) Determine $\tan^{-1}(0)$.

$\tan(\theta)=0$

$\frac{\sin(\theta)}{\cos(\theta)}=0$

$\cos(\theta)\left(\frac{\sin(\theta)}{\cos(\theta)}\right)=(0)\cos(\theta)$

$\sin(\theta)=0$

Possible $\theta=-2\pi,-\pi,0,\pi,2\pi$

Range of inverse tangent is $\left(\frac{-1}{2}\pi,\frac{1}{2}\pi\right)$

$\frac{-1}{2}\pi<0<\frac{1}{2}\pi$

A. $-\pi$ incorrect angle, outside range

B. 0

C. $\frac{1}{2}\pi$ incorrect angle, outside range

D. π outside range

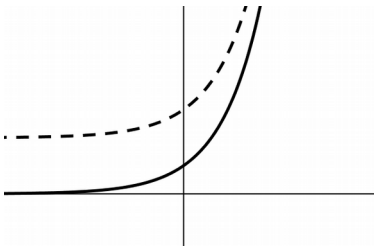
E. 2π outside range

18. (4 points) The graph of 2^{2x} is a transformation of the graph of 2^x. The solid line represents 2^x.

In which of the following graphs does the dotted line represent 2^{2x}?

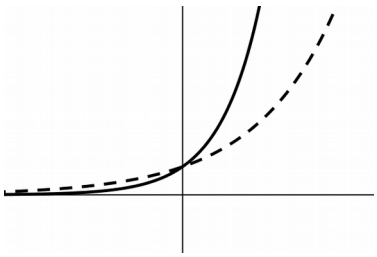
Horizontal shrink by a factor of 2

A.



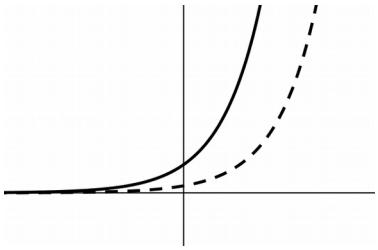
2^x+2

B.



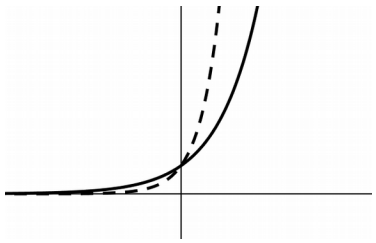
$2^{\frac{1}{2}x}$

C.

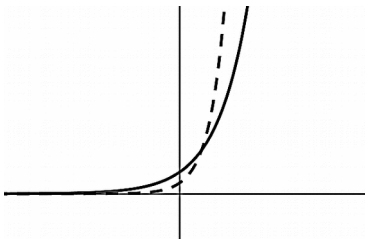


2^{x-2}

D.



E.

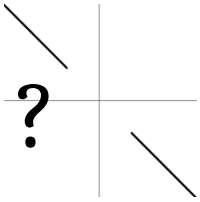


2^{2x-2}

19. (4 points) Which of the following is a possible graph of $f(x)=-3(x+1)^2(x-2)$?

Sum of exponents: $2-1=3$

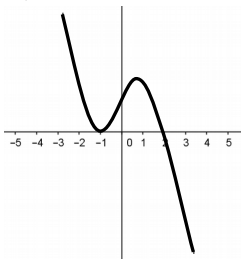
End behavior: $y=-x^{odd}$



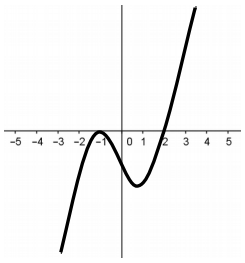
x intercepts: -1 with a multiplicity of 2 (bounce), 2 with a multiplicity of 1 (though)

y intercept: $f(0)=-3(0+1)^2(0-2)=-3(1)^2(-2)=6$

A.

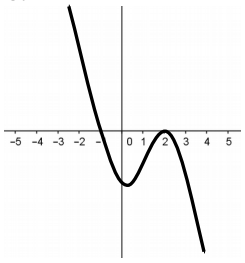


B.



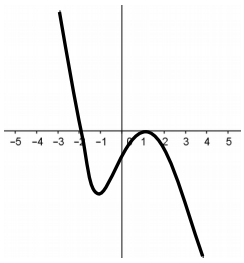
incorrect extreme behavior

C.



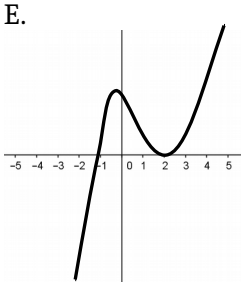
incorrect roots, incorrect extreme behavior

D.



incorrect roots

E.



incorrect extreme behavior, incorrect multiplicity behavior

20. (4 points) Simplify $\frac{3+3\sqrt{6}}{6}$ completely.

$$\frac{3(1+\sqrt{6})}{3(2)}=\frac{3(1+\sqrt{6})}{3(2)}=\frac{1+\sqrt{6}}{2}$$

A. $\frac{1+\sqrt{2}}{2}$ — 3 factored from inside $\sqrt{\square}$

B. $\frac{\sqrt{6}}{2}$ — leading term is 0

C. $\frac{1+\sqrt{6}}{2}$

D. $\frac{3\sqrt{6}}{2}$ — 3 factored from first term only, leading term is 0

E. $\frac{1+3\sqrt{6}}{2}$ — 3 factored from first term only

21. (4 points) Calculate $\frac{22}{21}-\frac{18}{35}$. Simplify your answer completely.

$21=3\cdot 7$

$35=5\cdot 7$

LCD ¿$3^1\cdot 5^1\cdot 7^1=3\cdot 5\cdot 7$

$$\frac{2\cdot 11}{3\cdot 7}-\frac{2\cdot 3\cdot 3}{5\cdot 7}=\frac{2\cdot 11}{3\cdot 7}\frac{(5)}{(5)}-\frac{2\cdot 3\cdot 3}{5\cdot 7}\frac{(3)}{(3)}=\frac{110}{3\cdot 5\cdot 7}-\frac{54}{3\cdot 5\cdot 7}=\frac{110-54}{3\cdot 5\cdot 7}=\frac{56}{3\cdot 5\cdot 7}=\frac{2\cdot 2\cdot 2\cdot 7}{3\cdot 5\cdot 7}=\frac{2\cdot 2\cdot 2}{3\cdot 5}=\frac{8}{15}$$

A. $\frac{56}{105}$ not simplified

B. $\frac{4}{105}$ only denominators multiplied to obtain LCD

C. $\frac{8}{15}$

D. $\frac{392}{735}$ LCD of 21×35, not simplified

E. $\frac{-2}{7}$ numerator and denominator subtracted

22. (4 points) Simplify $\log_a(a^x)$.

By the properties of logarithms, $\log_a(a^x)=x$

A. a^x

B. x^a

C. $\ln(x)$

D. x

E. $x\cdot a$

23. (4 points) In which quadrant(s) is the angle $\frac{17}{6}\pi$ located?

$$\frac{17}{6}\pi-2\pi=\frac{17}{6}\pi-\frac{12}{6}\pi=\frac{5}{6}\pi$$

A. I

B. II

C. III

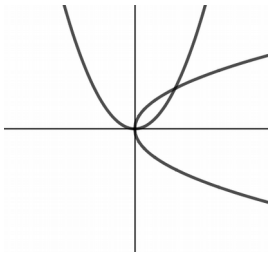
D. IV

E. $\frac{17}{6}\pi$ could be located in more than one quadrant

24. (4 points) Which of the following pairs of graphs is a function and its inverse function?

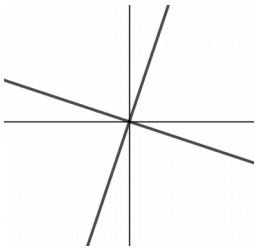
Must pass vertical line test and horizontal line test. Must be a reflection across the line $y=x$.

A.



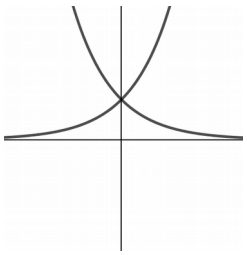
not a function

B.



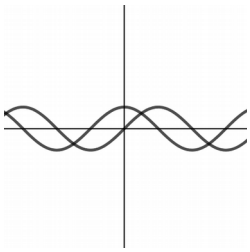
perpendicular

C.



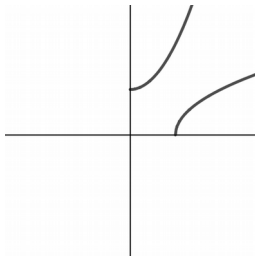
horizontal reflection

D.

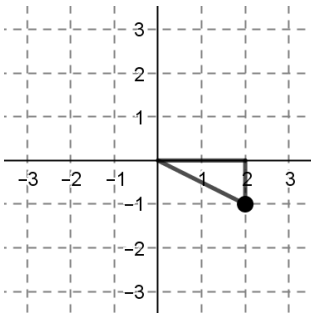


sine and cosine

E.



25. (4 points) A right triangle is composed of the origin, the x axis, and the point $(2,-1)$. Find the sine of the angle at the origin.



$a^2+b^2=c^2$

$2^2+(-1)^2=c^2$

$4+1=c^2$

$5=c^2$

$c=\sqrt{5}$

$\sin(\theta)=\frac{opp}{hyp}=\frac{-1}{\sqrt{5}}=\frac{-1}{\sqrt{5}}\frac{(\sqrt{5})}{(\sqrt{5})}=\frac{-\sqrt{5}}{5}$

A. $\frac{\sqrt{5}}{5}$ incorrect quadrant

B. $-\sqrt{5}$ hypotenuse/opposite

C. $\frac{-\sqrt{5}}{5}$

D. $\frac{2\sqrt{5}}{5}$ $\cos(\theta)$

E. $\frac{-\sqrt{5}}{2}$ hypotenuse/adjacent

26. (4 points) Determine the equation of the line containing the points $(1,4)$ and $(-3,7)$. Give your answer in slope intercept form.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{7-4}{-3-1}=\frac{3}{-4}=\frac{-3}{4}$$

$$y-y_1=m(x-x_1)$$

$$y-4=\frac{-3}{4}(x-1)$$

$$y-4=\frac{-3}{4}x+\frac{3}{4}$$

$$y=\frac{-3}{4}x+\frac{3}{4}+4=\frac{-3}{4}x+\frac{3}{4}+\frac{16}{4}=\frac{-3}{4}x+\frac{19}{4}$$

A. $y=\frac{-4}{3}x+\frac{16}{3}$ slope is $\frac{\Delta x}{\Delta y}$

B. $y=\frac{-4}{3}x+4$ y intercept from coordinates OR x and y coordinates of point switched

C. $y=\frac{-4}{3}x+7$ slope is $\frac{\Delta x}{\Delta y}$, y intercept from coordinates

D. $y=\frac{-3}{4}x+\frac{19}{4}$

E. $y=\frac{-3}{4}x+3$ $\frac{-3}{4}$ not distributed to -1

27. (4 points) Determine the domain of $f(x)=\ln(2-x)$. Give your answer in interval notation.

Cannot take log of 0 or a negative number: $2-x>0 \rightarrow 2>x$

A. $(-\infty,2)\cup(2,\infty)$ argument $\neq 0$

B. $(-\infty,2)$

C. $(2,\infty)$ incorrect half of line

D. $(-\infty,2]$ $\ln(0)$

E. $[-2,2]$ $|x|\leq 2$

28. (4 points) Find the distance between the point $(2,5)$ and the point $(3,-1)$.

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(3-2)^2+(-1-5)^2}=\sqrt{(1)^2+(-6)^2}=\sqrt{1+36}=\sqrt{37}$$

A. $\sqrt{37}$

B. $\sqrt{5}$ sum of differences

C. $\sqrt{41}$ sum of sums squared

D. 3 difference of sums squared

E. 7 sum of square roots

29. (4 points) Which of the following represents the characteristics of $-3\sin\left(\frac{1}{2}x\right)$?

Vertical stretch by a factor of 3 means amplitude ¿ 3

Horizontal stretch by a factor of 2 means period ¿ 4π

No horizontal shift means phase shift ¿ 0

A. amplitude ¿ 3, period ¿ 4π, phase shift ¿ 0

B. amplitude ¿ -3, period ¿ 4π, phase shift ¿ 0 — amplitude is negative

C. amplitude ¿ 3, period ¿ 2π, phase shift ¿ $\frac{1}{2}\pi$ — horizontal shift

D. amplitude ¿ 1, period ¿ 2π, phase shift ¿ 0 — incorrect amplitude and period

E. amplitude ¿ 1, period ¿ π, phase shift ¿ $\frac{1}{2}\pi$ — incorrect amplitude, horizontal shift

30. (4 points) Which of the following are true statements about e^x and $\ln(x)$?

Anything (except 0) to the 0 power is 1

Log base anything (greater than 0) of 1 is 0

A. $e^1=0, \ln(0)=1$ — $e^1\neq 0, \ln(0)\neq 1$

B. $e^1=0, \ln(1)=0$ — $e^1\neq 0$

C. $e^0=1, \ln(1)=e$ — $\ln(1)\neq e$

D. $e^0=1, \ln(0)=e$ — $\ln(0)\neq e$

E. $e^0=1, \ln(1)=0$

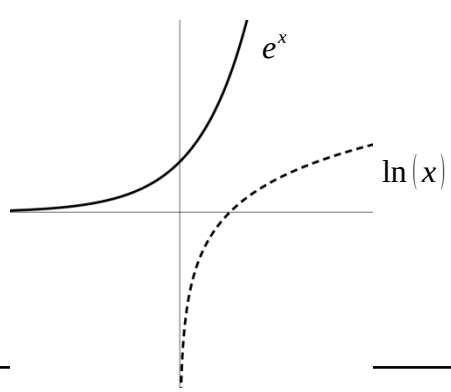
31. (10 points) Solve $2x^2+5x=12$ using two different methods. Simplify your answer(s) completely. Consider real solutions only.

$$2x^2+5x-12=0$$
$$(2x-3)(x+4)=0$$
$$2x-3=0 \quad x+4=0$$
$$2x=3 \qquad x=-4$$
$$x=\frac{3}{2}$$

$$2x^2+5x-12=0$$
$$x=\frac{-5\pm\sqrt{5^2-4(2)(112)}}{2(2)}=\frac{-5\pm\sqrt{25+96}}{4}=\frac{-5\pm\sqrt{121}}{4}=\frac{-5\pm 11}{4}=\frac{-5-11}{4},\frac{-5+11}{4}=\frac{-16}{4},\frac{6}{4}=-4,\frac{3}{2}$$

$$x^2+\frac{5}{2}x=6$$
$$x^2+\frac{5}{2}x+\left(\frac{5}{4}\right)^2=6+\left(\frac{5}{4}\right)^2$$
$$\left(x+\frac{5}{4}\right)^2=6+\frac{25}{16}$$
$$\left(x+\frac{5}{4}\right)^2=\frac{96}{16}+\frac{25}{16}$$
$$\left(x+\frac{5}{4}\right)^2=\frac{121}{16}$$
$$\sqrt{\left(x+\frac{5}{4}\right)^2}=\pm\sqrt{\frac{121}{16}}$$
$$x+\frac{5}{4}=\pm\frac{11}{4}$$
$$x=\frac{-5}{4}\pm\frac{11}{4}=\frac{-5}{4}-\frac{11}{4},-\frac{5}{4}+\frac{11}{4}=\frac{-16}{4},\frac{6}{4}=-4,\frac{3}{2}$$

32. (10 points) Sketch the graphs of e^x and $\ln(x)$ on the same set of axes. State the coordinates of any intercepts of each, the domain and range of each, and the **equations** of any asymptotes of each.



e^x coordinates of intercept(s):	$(0,1)$
e^x domain:	$(-\infty,\infty)$
e^x range:	$(0,\infty)$
e^x **equation(s)** of asymptote(s):	$y=0$

$\ln(x)$ coordinates of intercept(s):	$(1,0)$
$\ln(x)$ domain:	$(0,\infty)$
$\ln(x)$ range:	$(-\infty,\infty)$
$\ln(x)$ **equation(s)** of asymptote(s):	$x=0$

33. (10 points) Solve $3x^{\left(\frac{2}{3}\right)}=24$. Simplify your answer(s) completely. Consider real solutions only.

$$x^{\left(\frac{2}{3}\right)}=8$$
$$\left(x^{\left(\frac{2}{3}\right)}\right)^{\frac{3}{2}}=(8)^{\frac{3}{2}}$$
$$x^1=(8)^{\left(3\cdot\frac{1}{2}\right)}$$
$$x=\left((8)^3\right)^{\frac{1}{2}}$$
$$x=\pm\sqrt{8^3}$$
$$x=\pm\sqrt{8\cdot 8\cdot 8}$$
$$x=\pm\sqrt{2^3\cdot 2^3\cdot 2^3}$$
$$x=\pm\sqrt{2^9}$$
$$x=\pm 2^4\sqrt{2^1}$$
$$x=\pm 16\sqrt{2}$$

34. (5 points) The expression $-3(x-2)^2-5$ represents a parabola. Does the parabola have a maximum or a minimum? Explain how you know. What are the coordinates of the maximum or minimum? What are the domain and range of the parabola?

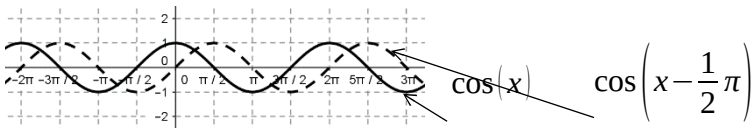
maximum or minimum:	maximum
because:	leading coefficient is negative
coordinates:	$(2,-5)$
domain:	$(-\infty,\infty)$
range:	$(-\infty,-5]$

35. (5 points) Solve $3e^{5x+1}=7$. Simplify your answer(s) completely. Consider real solutions only.

$$e^{5x+1}=\frac{7}{3}$$
$$\ln\left(e^{5x+1}\right)=\ln\left(\frac{7}{3}\right)$$
$$5x+1=\ln\left(\frac{7}{3}\right)$$
$$5x=\ln\left(\frac{7}{3}\right)-1$$

$$x=\frac{\ln\left(\frac{7}{3}\right)-1}{5}$$

36. (5 points) Accurately sketch at least two periods (cycles) of the graph of $\cos\left(x-\frac{1}{2}\pi\right)$. First show the base function, then show the transformation. List the amplitude, period, and phase shift.



amplitude: 1

period: 2π

phase shift: $\frac{1}{2}\pi$ right

37. (5 points) Solve $2\log_3(x+1)=4$. Simplify your answer(s) completely. Consider real solutions only.

$\log_3(x+1)=2$
$3^{\log_3(x+1)}=3^2$
$x+1=9$
$x=8$

OR

$\log_3\left((x+1)^2\right)=4$
$3^{\log_3\left((x+1)^2\right)}=3^4$
$(x+1)^2=81$
$\sqrt{(x+1)^2}=\pm\sqrt{81}$
$x+1=\pm 9$
$x=-1\pm 9=-1-9,-1+9=-10,8$
But $x=-10$ must be eliminated because it would result in the logarithm of a negative number
$x=8$

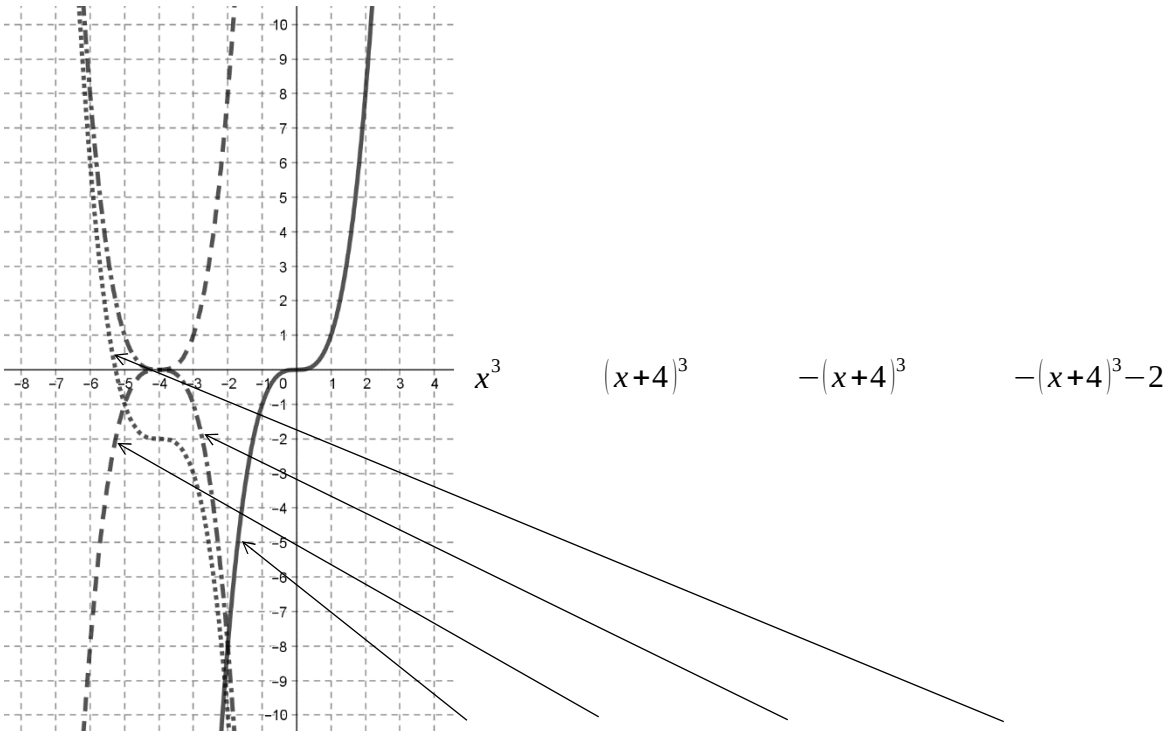
38. (5 points) Simplify the expression $\sqrt[3]{32x^3}\sqrt[3]{108x^5}$ completely. Give your answer with no rational (fractional) exponents.

$\sqrt[3]{32x^3\cdot 108x^5}=\sqrt[3]{2^5x^3 2^2 3^3 x^5}=\sqrt[3]{2^7 3^3 x^8}=2^2 3^1 x^2\sqrt[3]{2^1 x^2}=12x^2\sqrt[3]{2x^2}$

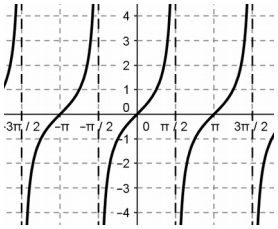
39. (5 points) Simplify the expression $\dfrac{x+1}{2-\dfrac{1}{x+1}}$ completely.

$$\frac{x+1}{\frac{2}{1}-\frac{1}{x+1}}=\frac{x+1}{\frac{2}{1}\frac{(x+1)}{(x+1)}-\frac{1}{x+1}}=\frac{x+1}{\frac{2x+2}{x+1}-\frac{1}{x+1}}=\frac{x+1}{\frac{2x+2-1}{x+1}}=\frac{x+1}{\frac{2x+1}{x+1}}=\frac{\frac{x+1}{1}}{\frac{2x+1}{x+1}}=\left(\frac{x+1}{1}\right)\left(\frac{x+1}{2x+1}\right)=\frac{x^2+x+x+1}{2x+1}=\frac{x^2}{}$$

40. (5 points) Accurately sketch the function $f(x)=-(x+4)^3-2$. First show the base function, then show the transformations. It may be helpful to create a table of x values and x^3 values.



41. (5 points) Accurately sketch at least two periods (cycles) of the graph of $\tan(x)$. List the **equations** of at least two of the asymptotes.



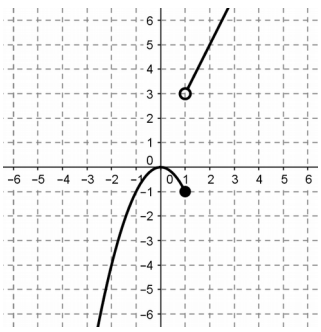
equations of asymptotes: $x = \ldots, -\frac{3}{2}\pi, -\frac{1}{2}\pi, \frac{1}{2}\pi, \frac{3}{2}\pi, \cdots$

42. (5 points) Solve $5x^5+15x^4-20x^3=0$ by factoring. Simplify your answer(s) completely. Consider real solutions only.

$5x^3(x^2+3x-4)=0$
$5x^3(x+4)(x-1)=0$

$5x^3=0$	$x+4=0$	$x-1=0$
$x^3=0$	$x=-4$	$x=1$
$x=0$		

43. (5 points) Accurately sketch the function $f(x)=\begin{cases} -x^2 & x\le 1 \\ 1+2x & x>1 \end{cases}$. Determine $f(1)$.



$f(1)=-1$

44. (5 points) Simplify the expression $\dfrac{-2x^2}{(3x)^{-3}}$ completely. Give your answer with no negative exponents and no rational (fractional) exponents.

$-2x^2(3x)^3=-2x^2 3^3 x^3=-54x^5$

45. (5 points) Solve $\dfrac{3}{4}-\dfrac{1}{6x}=\dfrac{1}{x}$. Simplify your answer(s) completely. Consider real solutions only.

$4=2^2$, $6x=2\cdot 3\cdot x$, $x=x$
LCD ¿$2^2\cdot 3\cdot x$

$$\frac{3}{2\cdot 2}\frac{(3\cdot x)}{(3\cdot x)}-\frac{1}{2\cdot 3\cdot x}\frac{(2)}{(2)}=\frac{1}{x}\frac{(2\cdot 2\cdot 3)}{(2\cdot 2\cdot 3)}$$

$$\frac{9x}{2\cdot 2\cdot 3\cdot x}-\frac{2}{2\cdot 2\cdot 3\cdot x}=\frac{12}{2\cdot 2\cdot 3\cdot x}$$

$$\frac{9x-2}{2\cdot 2\cdot 3\cdot x}=\frac{12}{2\cdot 2\cdot 3\cdot x}$$

$$(2\cdot 2\cdot 3\cdot x)\left(\frac{9x-2}{2\cdot 2\cdot 3\cdot x}\right)=\left(\frac{12}{2\cdot 2\cdot 3\cdot x}\right)(2\cdot 2\cdot 3\cdot x)$$
$$9x-2=12$$
$$9x=14$$
$$x=\frac{14}{9}$$

46. (5 points) Solve the system of equations $\begin{cases} 4x+3y=5 \\ 2x-y=2 \end{cases}$. Simplify your answer(s) completely. Consider real solutions only.

$$y=2x-2$$
$$4x+3(2x-2)=5$$
$$4x+6x-6=5$$
$$10x=11$$
$$x=\frac{11}{10}$$

$$y=2x-2=2\left(\frac{11}{10}\right)-2=\frac{22}{10}-2=\frac{22}{10}-\frac{20}{10}=\frac{2}{10}=\frac{1}{5}$$

OR

$$\begin{aligned} 4x+3y&=5 \\ 3(2x-y)&=(2)3 \\ 4x+3y&=5 \\ +6x-3y&=6 \\ \hline 10x&=11 \end{aligned}$$

Remaining solution as above

47. (5 points) Use long division to find $(2x^3-5x^2+5x-6)\div(x-2)$.

$$\begin{array}{r} 2x^2-x+3 \\ x-2\,\overline{)\,2x^3-5x^2+5x-6} \\ \underline{-2x^3+4x^2} \\ 0-x^2+5x \\ \underline{x^2-2x} \\ 0+3x-6 \\ \underline{-3x+6} \\ 0+0 \end{array}$$