NAME:___________________________________ ALPHA NUM:___________
INSTRUCTOR: ____________________________ SECTION: ______________

CALCULUS I (SM121, SM121A, SM131) **FINAL EXAMINATION**

0755-1055 Friday 18 December 2009 SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor, and section number on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS ARE PERMITTED FOR THIS MULTIPLE CHOICE SECTION.

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1. What is the DOMAIN of the function given by: $h(x) = \dfrac{1}{\sqrt{4-x^2}}$

a) $x \neq 2$ b) all reals c) $[-2,2]$ d) $(-2,2)$ e) $(0,\infty)$

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2. Which of the following equations has the graph of an EVEN function?
a) $y = \sin(x)$ b) $y = \cos(x)$ c) $y = e^x$ d) $y = \ln(x)$ e) $y = 1/x$

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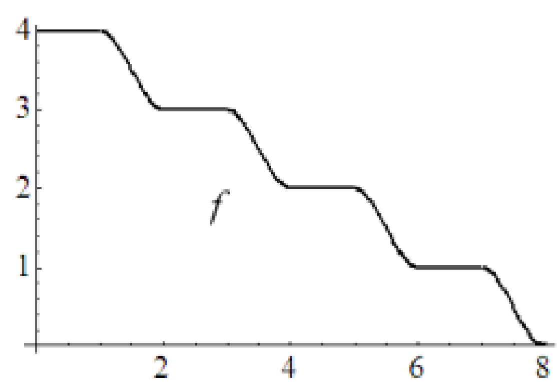
3. For f the function graphed and $g(x) = e^x$, find the value of the composition at 1, namely, $(f \circ g)(1) = f(g(1)) =$

a) 0 b) 1 c) 2 d) 3 e) e^4



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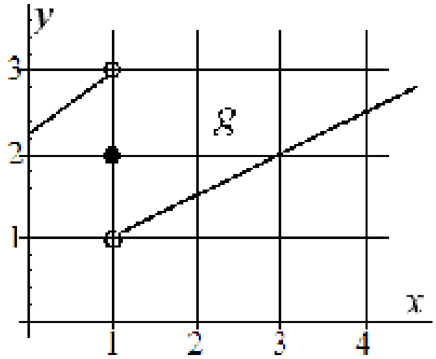
4. If $\lim_{x\to a} f(x) = 3$ and $\lim_{x\to a} g(x) = 0$ and $\lim_{x\to a} h(x) = 1$, then $\lim\limits_{x\to a}(f(x)g(x) + h(x)) =$

a) 0 b) 1 c) 2 d) 3 e) not enough information to evaluate

5. Based on the given graph of g, the left-hand limit as x approaches 1 from the left hand side is given by
$\lim_{x\to 1^-} g(x) =$



a) 0 b) 1 c) 2 d) 3 e) does not exist

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6. Which equation is of a horizontal asymptote to the graph of
$y = \dfrac{2x^2+1}{x^2-4x+3}$

a) $x = 1$ b) $x = 3$ c) $y = 0$ d) $y = 1$ e) $y = 2$

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7. Suppose that the table shows the percentage P of the population in Europe that uses cell phones in each year.

Year	2000	2001	2002	2003
P	52	61	73	82

What is the average rate of cell phone growth from 2000 to 2003? (Units are percent/yr.)

a) 10 b) 15 c) 30 d) 82 e) none of these

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8. Which is an equation of the tangent line to $y = x^2 - 2x^3$ at $(-1,3)$?

a) $y - 3 = (2x - 6x^2)(x+1)$ b) $y - 3 = -8(x+1)$ c) $y = -8x$

d) $y + 1 = -8(x-3)$ e) none of these

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9. If $f(4) = 2$ and $f'(4) = 3$ and $g(x) = \sqrt{x}$, then the derivative of the product at 4 is $(fg)'(4) =$

a) 4 b) 6 c) 8 d) $\frac{11}{2}$ e) $\frac{13}{2}$

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10. Evaluate: $\frac{d}{dx}\sec(2x) =$

a) $\tan^2(2x)$ b) $2\tan^2(2x)$ c) $\sec(2x)\tan(2x)$

d) $2\sec(2x)\tan(2x)$ e) $4\sec(2x)\tan(2x)$

11. Differentiating $x/y = \sin(x-y)$ implicitly with respect to x gives:

a) $\frac{y-xy'}{y^2} = (1-y')\cos(x-y)$ b) $\frac{y-xy'}{y^2} = y'\cos(x-y)$

c) $\frac{1}{y'} = (1-y')\cos(x-y)$ d) $\frac{y-xy'}{y^2} = \cos(x-y)$ e) $\frac{1}{y'} = \cos(x-y)$

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12. A spherical balloon is being inflated. Find the rate of increase of the surface area ($S = 4\pi r^2$) with respect to time when the radius r is 3cm if $\frac{dr}{dt} = 2$cm/s.

a) 6πcm^2/s b) 8πcm^2/s c) 24πcm^2/s d) 48πcm^2/s e) 72πcm^2/s

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13. The linearization of $\ln(x)$ at $a = 1$ is

a) $L(x) = x$ b) $L(x) = \frac{1}{x}(x-1)$ c) $L(x) = x - 1$ d) $L(x) = \frac{1}{x} - 1$ e) undefined

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14. On the interval $[-1,2]$ the absolute maximum of $h(x) = x^2$ is:

a) -1 b) 0 c) 1 d) 2 e) 4

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15. For $f(x) = e^{-2x}$ on the interval $[0,2]$ the value of c that satisfies the conclusion of the Mean Value Theorem is closest to

a) 0 b) 0.7 c) 1.0 d) 1.3 e) 2.0

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16. Suppose that f is continuous everywhere and that $f'(x) = 0$ for $x = 0, 3$, and 5. Further, suppose $f'(x) > 0$ for $x < 0$, $3 < x < 5$, and $5 < x$. Also, $f'(x) < 0$ for $0 < x < 3$. Then f has a local minimum at $x =$

a) 0 b) 3 c) 4 d) 5 e) not enough information

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17. Suppose $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} f'(x) = 3$ and $g(x) = \sin(2x)$. Then $\lim_{x\to 0} \frac{f(x)}{g(x)} =$

a) 0 b) $^{1}/_{2}$ c) 1 d) $^{3}/_{2}$ e) 3

18. Find the most general expression for function h if the second derivative is given by $h''(x) = e^x$:

a) $h(x) = e^x$ b) $h(x) = -e^{-x}$ c) $h(x) = Ce^x + D$

d) $h(x) = e^x + Cx + C$ e) $h(x) = e^x + Cx + D$

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19. The velocity of a plane traveling in a straight line is given at 12 second intervals:

t (s)	0	12	24	36	48	60
v (m/s)	100	110	115	115	120	125

Estimating the distance traveled during this minute of time using the velocities at the beginning of the time intervals gives:

a) 6720 m b) 6795 m c) 6870 m d) 6945 m e) 7020 m

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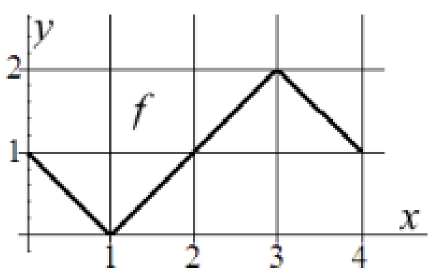
20. For f as sketched, and g defined by

$g(x) = \int_0^x f(t)dt$

what is $g(3)$?

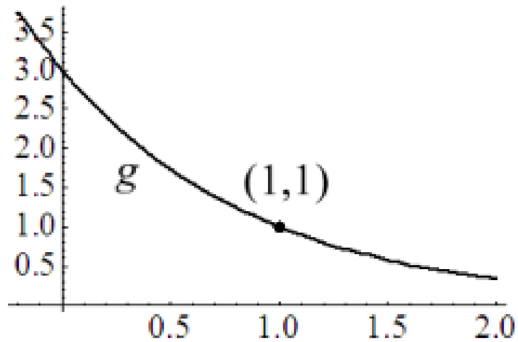
a) 2 b) 2.5 c) 3 d) 3.5 e) 4



Part Two. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES. **CALCULATORS PERMITTED FOR ALL PROBLEMS <u>EXCEPT</u> 27, 28, 29, 30.**

21. a) Find the exponential function $g(x) = Ca^x$ with the given graph.

b) Give an equation for the tangent line to the graph of g at $x = 0$.

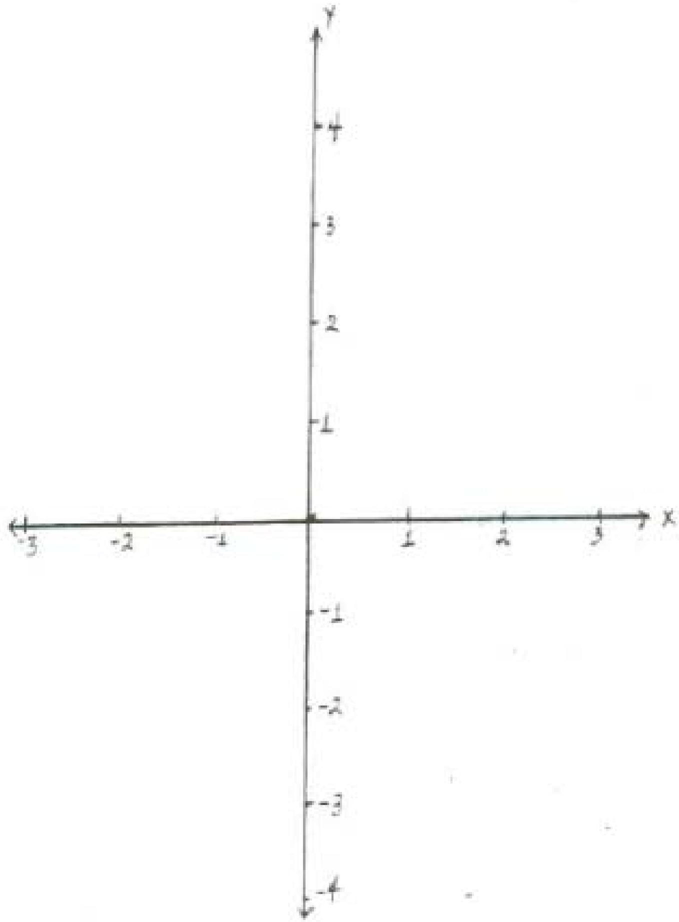


22. Find a formula for $f^{-1}(x)$, the inverse of the function given by $f(x) = \sqrt{6 - 2x}$. What is the domain of f^{-1}?

23. Sketch the graph of a single function h that satisfies:

$\lim_{x \to -\infty} h(x) = -1$, $\lim_{x \to -2^-} h(x) = 1$, $\lim_{x \to -2^+} h(x) = -\infty$,

$\lim_{x \to -1} h(x) = 0$, $\lim_{x \to 0} h(x) = -\infty$, $\lim_{x \to 2} h(x) = \infty$, $\lim_{x \to \infty} h(x) = 2$



24. Showing all steps, use logarithmic differentiation to find $\frac{dy}{dx}$ for

$y = (\tan(x))^{1/x}$

25. At noon ship A is 60 km north of location X and sailing north at 10 km/hr. At the same time ship B is 80 km east of X and sailing WEST at 13 km/hr. At what rate is the distance between the ships changing?

26. Find the point on the curve with equation $y = \sqrt{x}$ that is closest to the point $(2,0)$. [*Hint*. You might try minimizing d^2, the square of the distance from a point of the curve to $(2,0)$.]

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CALCULATORS NOT PERMITTED FOR 27, 28, 29, 30.

27. Use the Chain Rule and the Product Rule to give a proof of the Quotient Rule.
[*Hint:* Write $\frac{f(x)}{g(x)} = f(x)[g(x)]^{-1}$.]

28. Find the derivative $r'(t)$ if $r(t) = \ln(t) + \tan^{-1}(2t) + 3t\sin(t) + \cos(3t) - 2e^{-t}$.

29. Sketch the graph of $y = x^4 - 4x^3$ and give:

a) Open intervals of increase

b) Local minimum and local maximum values

c) Open intervals of upward concavity

d) Points of inflection

30. Find the general indefinite integral: $\int (x^2 + x^{-2} + \sin(x))\, dx$