Name _____ Alpha _____ Section_____

SPRING 2009 FINAL EXAMINATION FOR CALCULUS 1, SM121A

4 May 2009 - 1330 - 1630

Part One - No Calculator - 30% - You may not use your calculator on this portion of the exam. When you have finished this part of the exam, hand it in and get the rest of the exam. All work and all answers should be shown on the exam itself.

1. Give the following values.
   a. $\sin(-\frac{\pi}{2})$

   b. $\tan^{-1}(\sqrt{3})$

   c. $\ln(1)$

2. Find the following limits. Express your answers as a finite number, $+\infty$, $-\infty$, or DNE (does not exist).
   a. $\lim_{x\to 2} \frac{x-2}{x^2-4}$

   b. $\lim_{x\to \infty} \frac{x-2}{x^2-4}$

   c. $\lim_{t\to 0} \frac{e^{3t}-1}{e^t-1}$

   d. $\lim_{x\to 1^+} \frac{x}{1-x}$

3. Differentiate the following functions.
   a. $f(t) = 4\cos(5t^2)$

b. $g(x) = \frac{x \ln x}{\sin x}$

4. Find the absolute maximum and absolute minimum of $f(x) = x^3 - 6x^2 + 1$ on the interval $[-1, 2]$.

5. Find $f(x)$ if $f'(x) = \frac{2}{x^3} - 1$ and $f(1) = 3$.

6. Find $\int 2t(t+2)\, dt$

7. Compute $\int_1^4 (x^2 - \sqrt{x})\, dx$.

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Part Two - Calculator Allowed - 40% - Multiple Choice - The next 15 problems are multiple choice. Fill in the best answer on the bubble sheet. There is no penalty for a wrong answer.

Write your name, alpha number and section at the top of this page and on the bubble sheet. ***Carefully bubble in your alpha number*** in the left-most columns of the bubble sheet.

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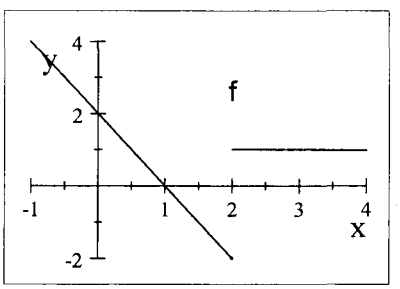
1. If $\cos\theta = -\frac{1}{2}$ and $\pi < \theta < 2\pi$, then $\theta =$

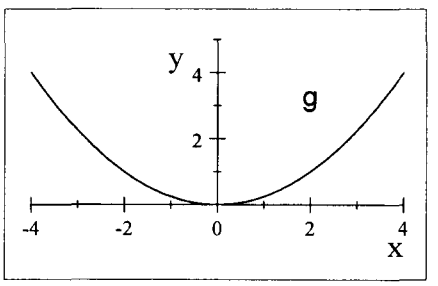
(a) $\frac{2\pi}{3}$ (b) $\frac{7\pi}{6}$ (c) $\frac{4\pi}{3}$ (d) $\frac{11\pi}{6}$ (e) $\frac{7\pi}{3}$

2. The inverse of the function $y = 3e^{x-1}$ is

(a) $y = 1 + \ln x - \ln 3$ (b) $y = \frac{1}{3}\ln(x+1)$ (c) $y = \frac{1}{3}e^{x+1}$ (d) $y = \frac{1}{3}e^{1-x}$ (e) $y = \frac{\ln x}{\ln 3} + 1$

Graphs for problems 3, 4, 5 and 6





3. $g(f(0)) =$
(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

4. $\lim_{x\to 2} f(x) =$
(a) 0 (b) −2 (c) $-\frac{1}{2}$ (d) 1 (e) does not exist

5. $f'(1) =$
(a) −2 (b) −1 (c) 0 (d) 1 (e) 2

6. $\lim_{x\to 1} \frac{f(x)}{3x^2-3} =$
(a) −2 (b) $-\frac{1}{3}$ (c) 0 (d) $\frac{1}{3}$ (e) 1

The table below is to be used with questions 7 and 8.

x	1	2	3	4	5	6
$f(x)$	3	4	6	5	2	1
$f'(x)$	1	2	0.5	−2	−2	−1
$g(x)$	4	3	2	−1	2	1
$g'(x)$	−1	−1	−2	0	1	−2

7. If $k(x) = f(x)g(x)$, then $k'(2) =$
(a) −2 (b) −1 (c) 0 (d) 1 (e) 2

8. If $h(x) = g(f(x))$, then $h'(3) =$
(a) −2 (b) −1 (c) 0 (d) 1 (e) 2

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9. The equation of the tangent line to $y = x^3$ at the point $(2,8)$ is

(a) $y - 8 = 3x^2(x-2)$ (b) $y = 12x - 16$ (c) $y = 12x + 16$ (d) $y = 12x$ (e) $y = 8$

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10. The function $g(x) = \begin{cases} \sin x, & \text{if } x \le 0 \\ x, & \text{if } 0 < x < 1 \\ 5x + 2, & \text{if } 1 \le x \end{cases}$ is continuous except at

(a) $x = 0$ (b) $x = \frac{1}{2}$ (c) $x = 1$ (d) $x = 0$ and $x = 1$ (e) it is continuous everywhere

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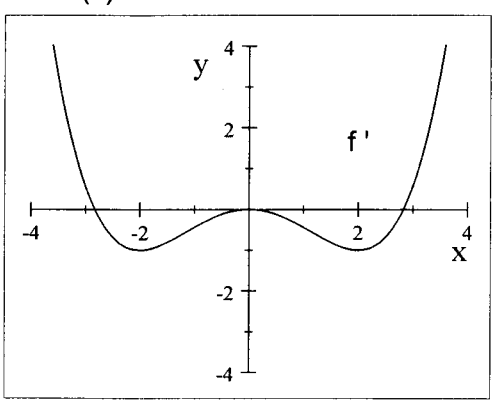
11. If the graph of f', the first derivative of f, is as shown below, then the function f has a local minimum at what x value(s)?



(a) $x = -2.8$ (b) $x = -2$ and $x = 2$ (c) $x = 0$ (d) $x = -2$, $x = 0$ and $x = 2$
(e) $x = 2.8$

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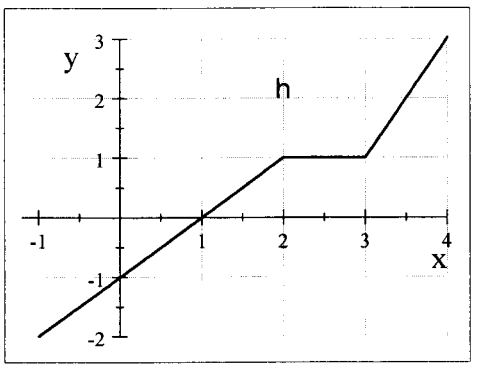
12. The largest interval on which the function $f(x) = -x^2 + 2x + 3$ is concave downward is
(a) $(-1,1)$ (b) on $(-\infty,1)$ (c) on $(1,\infty)$ (d) $(-\infty,\infty)$ (e) $(0,\infty)$

13. The graph of the function h is given below.



Then $\int_{-1}^{3} h(x)\,dx =$

(a) 3.5 (b) 2.5 (c) 1.5 (d) 0.5 (e) -0.5

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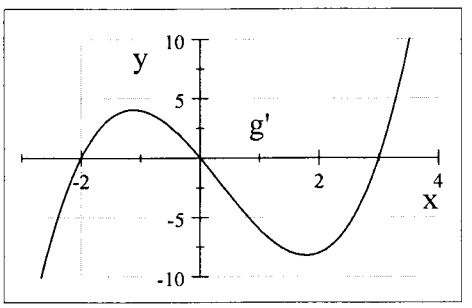
14. The graph of the **first derivative** g' of g is given below.



The function g is concave downward on

(a) $(-3,-1.1)$ and $(1.7,4)$ (b) $(-3,0)$ (c) $(-1.1,1.7)$ (d) $(-2,0)$ and $(3,4)$ (e) $(-3,-2)$ and $(0,3)$

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15. If the area between the x-axis and the graph of $f(x) = 1 + x^2$ for $1 \le x \le 5$ is approximated by M_2, the midpoint rule with two rectangles, the answer is

(a) 22 (b) 24 (c) 36 (d) 44 (e) 72

Part Three - Calculator Allowed - 30% - Longer Answer - Work the following problems. They are not multiple choice. Show all work and put all answers in the space provided on the test itself.

16. Given the following information about the function g,

$$\lim_{x \to -\infty} g(x) = 2, \quad g(0) = 6, \quad g(4) = 0$$

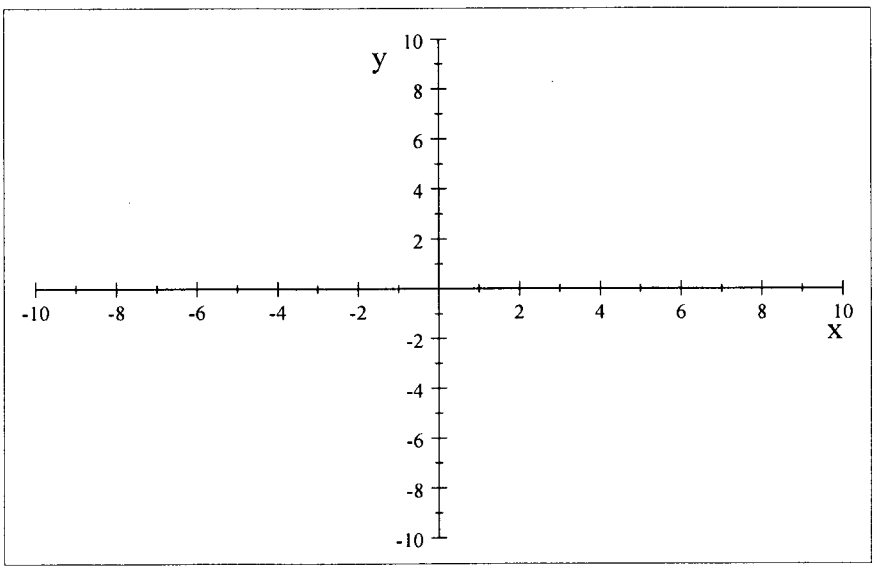
$$g'(x) > 0 \text{ on } (-\infty, 0) \text{ and } (4, \infty)$$

$$g'(x) < 0 \text{ on } (0, 4)$$

$$g''(x) > 0 \text{ on } (-\infty, -2) \text{ and } (2, \infty)$$

$$g''(x) < 0 \text{ on } (-2, 2)$$

sketch the graph of g. Be sure to label all the important points.



17. Use the definition of derivative to find $f'(x)$ if $f(x) = 3x^2 - 2x$. You must use the definition to receive any credit for this problem.

18. Consider the curve given by the equation

$$-x + x^3y^2 + 6y = 12.$$

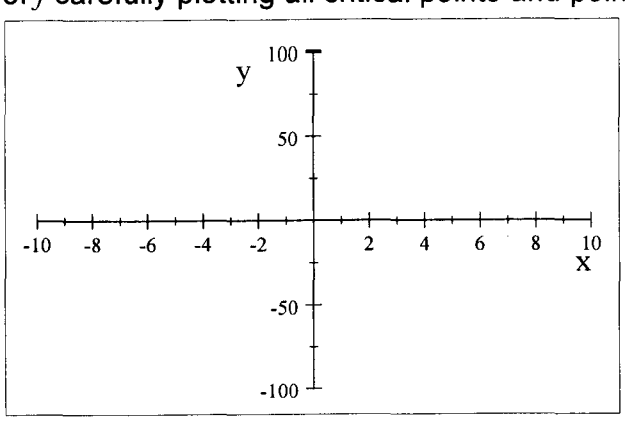
Find the slope of the tangent line to the curve at the point $(2, 1)$.

19. Let $f(x) = x^4 - 18x^2 + 1$.

a. On what intervals is f increasing? decreasing?

b. On what intervals is f concave upward? downward?

c. Sketch the graph of f carefully plotting all critical points and points of inflection.



20. A baseball diamond is a square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s. At what rate is his distance from second base decreasing when he is halfway to first base?

21. Midn Wolf is building a rectangular box with square base and open top. He is going to paint the outside of the box a combination of blue and gold and he has enough paint to cover 200 square inches. What is the largest possible volume of the box?

22. Midn Wolf throws a ball upward from a height of 200 feet with initial speed of 32 feet per second. Recall that acceleration due to gravity is $-32 ft/s^2$.

a. Find its height $s(t)$ after t seconds.

b. Midn Fox is standing in the path of the ball. If Midn Fox is exactly 6 ft tall, when does the ball hit the top of his head?

c. With what velocity does the ball hit Midn Fox?