

NAME: \_\_\_\_\_

ALPHA \_\_\_\_\_

INSTRUCTOR: \_\_\_\_\_

SECTION \_\_\_\_\_

**NO CALCULATORS ALLOWED ON THIS PART** SHOW ALL WORK IN THIS EXAM PACKAGE

This exam consists of three parts. You must do all work on the exam package and turn in all parts.

Part I multiple choice without calculator ( approx. 50% ... 17 problems, 3% each)

Part II long answer without calculator (approx. 25% ... 4 problems, 6 % each)

Part III long answer with calculator (approx. 25%... 4 problems, 6 % each)

1. Write your name, alpha number, instructor, and section number on this test and your Scantron bubble sheet. Bubble in your alpha number and the **VERSION NUMBER** of Part I of your exam.
2. **Do Parts I and II first. No calculators are allowed for Parts I and II.** Show all work on your exam packet. You must turn in Parts I and II of your exam, including bubble sheets, before using a calculator. Parts I, II and the bubble sheet are due **20 min prior to the end of the exam period.**
3. Calculators are allowed for Part III. Calculators may not be shared. Show all work on your exam packet.
4. There is a list of formulas on the last page of Part III which you may use for any part of the exam.
5. Page 13 is intentionally left blank, in case you need extra paper to show your work.

PART I: MULTIPLE CHOICE (50%). NO CALCULATORS ALLOWED. Do all work on the exam packet. You must turn in Parts I and II of the exam, including bubble sheets, before using a calculator, and no later than **20 min prior to the end of the exam period.** Be sure that you have put your name, alpha number, instructor, and section number on your bubble sheet and bubbled in your alpha number and VERSION number. Fill in the answer to each question on your Scantron bubble sheet. There is no extra penalty for wrong answers on the multiple choice part of the exam.

1. The lines  $y = 2x$  and  $y = \frac{2}{3}(x + 24)$  intersect at a point  $P$ . The  $x$  coordinate of  $P$  is in the interval:
  - a)  $(-\infty, 10]$
  - b)  $(-10, 5]$
  - c)  $(5, 10]$
  - d)  $(10, 15]$
  - e)  $(15, \infty)$

2. Simplify the fraction  $F$  given below. What does  $F$  equal to?

$$F = \frac{a^{-15}b^8}{a^3b^4} \times \frac{\frac{3}{5}}{15}.$$

- a)  $9a^{-5}b^2$
- b)  $\frac{b^2}{25a^5}$
- c)  $\frac{\sqrt[5]{a} b^2}{25}$
- d)  $\frac{b^4}{25a^{18}}$
- e)  $\frac{9 b^4}{a^{18}}$

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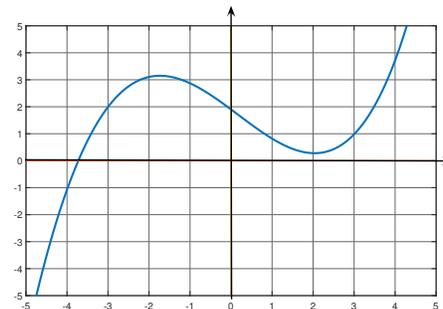
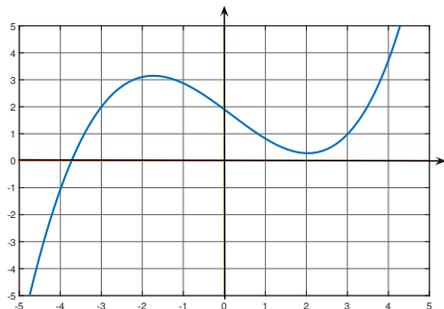
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3. Kathy is a repair technician for a phone company. Each week, she receives a batch of phones that need repairs. The number of phones that she has left to fix at the end of each day can be estimated with the equation  $P(d) = 108 - 23d$ , where  $P$  is the number of phones left and  $d$  is the number of days she has worked that week.

What is the meaning of the value 108 in this equation?

- a) Kathy will complete the repairs within 108 days.
- b) Kathy starts each week with 108 phones to fix.
- c) Kathy repairs phones at a rate of 108 per hour.
- d) Kathy repairs phones at a rate of 108 per day.
- e) Kathy will need 108 hours to complete the repairs.

4. Given the graph of the function  $f$  as shown, which of the following are arranged in increasing order? (Duplicate graphs are provided for convenience.)



- a)  $f'(-4)$ ,  $f'(2)$ ,  $\frac{f(3)-f(-4)}{3-(-4)}$ ,  $f'(3)$ ,  $f'(0)$
- b)  $f'(0)$ ,  $f'(2)$ ,  $\frac{f(3)-f(-4)}{3-(-4)}$ ,  $f'(3)$ ,  $f'(-4)$
- c)  $f'(0)$ ,  $f'(2)$ ,  $f'(3)$ ,  $f'(-4)$ ,  $\frac{f(3)-f(-4)}{3-(-4)}$
- d)  $f'(-4)$ ,  $f'(0)$ ,  $f'(2)$ ,  $f'(3)$ ,  $\frac{f(3)-f(-4)}{3-(-4)}$
- e)  $\frac{f(3)-f(-4)}{3-(-4)}$ ,  $f'(3)$ ,  $f'(2)$ ,  $f'(0)$ ,  $f'(-4)$

5. Given the graph of the function  $f$  from problem 4 (shown above), find an approximate value for the derivative of  $G(x) = \ln(f(x))$  at  $x = 0$ :

- a)  $\frac{dG}{dx}(0) = \frac{-1}{2}$
- b)  $\frac{dG}{dx}(0) = -1$
- c)  $\frac{dG}{dx}(0) = e^{-3.7}$
- d)  $G$  and  $\frac{dG}{dx}$  are undefined at  $x = 0$
- e)  $\frac{dG}{dx}(0) = \frac{1}{2}$

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6. For  $f(x) = e^x$ , after simplifying the ratio  $\frac{f(x+h)-f(x)}{h}$ , one gets:

- a)  $e^x$
- b) 1
- c)  $\frac{e^x(e^h-1)}{h}$
- d)  $\frac{e^h}{h}$
- e)  $\frac{e^{1+h/x}}{h}$

7. If  $\theta$  is an angle in the interval  $[0, \pi/2]$  with  $\tan \theta = \frac{\sqrt{5}}{2}$ , then the following is true about the angle:

- a)  $\sin \theta = \frac{\sqrt{5}}{3}$
- b)  $\sin \theta = \frac{2}{\sqrt{5}}$
- c)  $\sin\left(\frac{\sqrt{5}}{2}\right) = \frac{\sqrt{5}}{3}$
- d)  $\theta = \sin\left(\frac{\sqrt{5}}{3}\right)$
- e)  $\theta = \sin\left(\frac{2}{\sqrt{5}}\right)$

8. S. O’Nelli is a talented basketball player, who was already 2.08 meters tall, shoe size 16, and 96 kilograms by the time he entered highschool. Let  $F(t)$  be the length of S. O’Nelli’s foot, in centimeters,  $t$  years after he was born in 1975. Which of the following sentences best describes the meaning of the equation  $F'(15) = 0.8$ ?

- a) When he was size 16 he was over 0.8.
- b) For 15 years he gained 0.8 kilograms a year.
- c) On his 15th birthday, his foot grew about 0.8cm.
- d) During the year he was 15, he gained about 0.8cm in foot length.
- e) For 15 years his foot gained 0.8 centimeters.

9. The derivative of the function  $y = x \sin(\pi x)$  is:

- a)  $\cos(\pi x)$
- b)  $-\cos(\pi x)$
- c)  $\sin(\pi x) + x\pi \cos(\pi x)$
- d)  $\frac{1}{2}x^2 \cos(\pi x)$ .
- e)  $\pi \cos(\pi x)$

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10. Consider the graphs of following functions, where  $x$  is in the interval  $(0, \infty)$ :

$$L(x) = \ln x, \quad M(x) = \frac{1}{x}, \quad P(x) = x^4, \quad R(x) = \sqrt{x}, \quad S(x) = \sin x, \quad T(x) = \tan x.$$

Among the functions  $L, M, P, R, S, T$  those that are increasing and concave down on  $(0, \infty)$  are (list all of them):

- a)  $L$  and  $R$
- b)  $L, R$  and  $S$
- c)  $M, R$  and  $T$
- d)  $P$  and  $T$
- e)  $M$

11. Recent studies indicate that the (average) surface temperature of the earth has been rising steadily. Some scientists have modeled the temperature by the linear function  $T = 0.02t + 8.50$ , where  $T$  is the temperature in degrees Celsius and  $t$  represents years since 1900. What does the slope represent?

- a) In 1900, the slope is the tangent line to the surface temperature.
- b) Since 1900, the surface temperature has been increasing by about 0.02 degrees Celsius per year.
- c) The slope of the surface temperature was 8.50 in 1900, and increasing by 0.02.
- d) The slope is where the tangent line becomes the linearization of  $f(x)$ .
- e) Since 1900, the warming of the earth surface is slowing down at a slope of  $\frac{8.50}{0.02}$  degrees Celsius per year.

12. Compute the limit  $L = \lim_{x \rightarrow -\infty} \frac{\ln(e^x + 49)}{4^x + 2}$ . What does it say about the function  $f(x) = \frac{\ln(e^x + 49)}{4^x + 2}$ ?

- a)  $L = \ln 7$ ; the function  $f$  has  $y = \ln 7$  as a horizontal asymptote
- b)  $L = \frac{\ln 50}{3}$ ; the function  $f$  has  $\ln \sqrt[3]{50}$  as a horizontal asymptote
- c)  $L$  does not exist; the function  $f$  has no horizontal asymptotes
- d)  $L$  does not exist; the function  $f$  has a vertical asymptote
- e)  $L = 0$ , the function has a vertical asymptote at  $x = 0$ .

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13. The position  $s$ , of a particle as a function of time  $t$ , is given by  $s = 200 \ln t + \frac{t^3}{2}$ . Position is in meters, time in seconds. Compute the velocity and the acceleration of the particle at time  $t = 2$ . What do they tell you about the particle at time 2?

- a) The particle has speed of  $106m/s$ , and deceleration of  $44m/s^2$ .
- b) The particle has speed of  $106m/s$ , and acceleration of  $206m/s^2$ .
- c) The particle has speed of  $112m/s$ , and deceleration of  $62m/s^2$ .
- d) The speed is  $4 + 200 \ln 2$ , the acceleration of the particle is undefined at time 2.
- e) The speed and the acceleration of the particle are undefined at time 2.

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14. Find the x-coordinate for the local minima and local maxima for the function  $F(x) = e^{2x} - 6x$ .

- a) local max at  $x = \ln 6$ ; no local min
- b) local max at  $x = 0$ ; local min at  $x = \ln 6$
- c) local min at  $x = 0$ ; local max at  $x = \ln 6$
- d) local min at  $x = 0.5 \ln 3$ ; local max at  $x = \ln 6$
- e) local min at  $x = 0.5 \ln 3$ ; no local max

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15. A curve is given by the equation  $9x + y \sin(\pi x) = y^2$ . Use implicit differentiation to find the value of  $\frac{dy}{dx}$  at the point  $x = 1, y = 3$ .

- a) 3
  - b)  $\frac{3-\pi}{2}$
  - c)  $\frac{9}{\pi+2}$
  - d)  $\frac{9-\pi}{6}$
  - e)  $y = 3 + 3(x - 1)$
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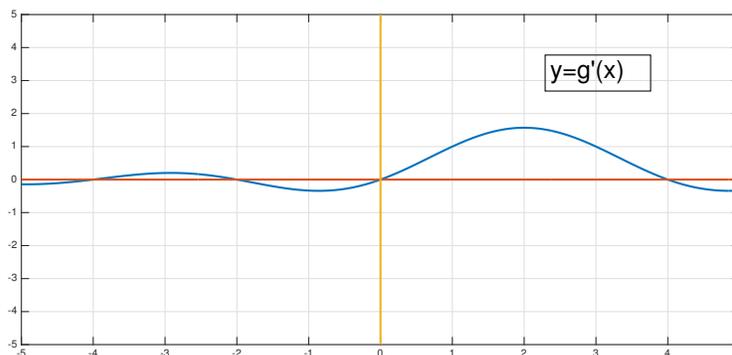
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16. An electrical circuit built using a battery of voltage  $V = 40$  volts and a resistance  $R = 400$  ohms creates a current  $I = \frac{1}{10}$  amps. As the battery wears out, the voltage  $V$  starts decreasing, and the resistance  $R$  starts increasing (the resistor heats up). Use  $V = IR$ , (Ohm's Law) to find the initial rate of change for the current  $I$  if  $\frac{dV}{dt} = -1$  volts per minute and  $\frac{dR}{dt} = 5$  ohms per minute.

- a)  $-\frac{3}{800}$  amps per minute
- b)  $-\frac{1}{5}$  amps per minute
- c)  $\frac{1}{5}$  amps per minute
- d) 5 amps per minute
- e)  $\frac{800}{3}$  amps per minute

17. The graph of the derivative  $g'$  of a differentiable function  $g$  is shown. On what intervals is  $g$  increasing?



- a)  $[-5, -4]$ ,  $[-2, 0]$ , and  $[4, 5]$ .
- b)  $[-5, -3]$  and  $[-1, 2]$ .
- c)  $[-5, -4]$ ,  $[-2, 1]$ , and  $[4, 5]$ .
- d)  $[-4, -2]$ , and  $[0, 4]$ .
- e) The function  $g$  is never increasing

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PART II: LONG ANSWER. NO CALCULATORS. You must turn in Parts I and II, and the bubble sheet before using your calculator. Parts I,II and the bubble sheet are due **20 min prior to the end of the exam period.**

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18. Let  $f(x) = 4 \ln(x + 3)$ .

- a. State the domain of  $f$  and the equation of its vertical asymptote.
- b. Find the  $x$  and  $y$  intercepts for the graph of  $f$ .
- c. Sketch the graph of  $f$ ; incorporate the results from a,b in your graph.
- d. Find the inverse function of  $f$ .
- e. On a new coordinate system, graph both  $f$  and  $f^{-1}$ ; make sure to label at least one intercept and one asymptote for  $f^{-1}$ .

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19. a) State the limit definition of the derivative.
- b) Use the definition of the derivative to show that  $\frac{d}{dx}(2x^2 + x) = 4x + 1$ .

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20. State and prove ONE of the following: Option 1: Product rule for computing the derivative of  $f(x)g(x)$ .  
Option 2: The derivative of  $y = \sin x$  is  $\frac{dy}{dx} = \dots$

Note: You may benefit from looking up the results listed on page 14.

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21. Consider the function  $f(x) = \sqrt[3]{x+8}$ .a. Find the equation for the tangent line to  $y = f(x)$  at  $a = 0$ .b. Use your linear approximation from a. to estimate  $\sqrt[3]{8.24}$ .c. Sketch the graph of  $f$  (mark the intercepts on the graph), and the graph of the tangent line from a.

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**YOU SHOULD TURN IN PARTS I, II, and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART**

Show details of your work and box your answers.

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PART III: LONG ANSWER. Calculators allowed AFTER TURNING IN PARTS I, II and the SCANTRON FORM. Parts I, II and the bubble sheet are due **20 min prior to the end of the exam period.**

22. Consider the function  $f(x) = x^2 + \frac{25}{x^2+1}$

a. Use the first derivative test to identify the intervals where  $f$  is increasing or decreasing. If you use the calculator to assist your computations, make sure to show the relevant results.

b. Find the  $x$  and  $y$  coordinates of the local minima and local maxima for  $f$

c. Use the formula for  $f$  to determine whether  $f$  is even, odd, or neither.

d. Sketch  $f$  (incorporate your results from a,b,c into your sketch).

e. Find the most general antiderivative for  $f$ .

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Show details of your work and box your answers.

23. a. Find (show all work)  $\lim_{x \rightarrow \pi} f(x)$ , where

$$f(x) = \begin{cases} \frac{\sin(x)}{1+\cos x} & \text{for } 0 < x \leq 2, \\ \frac{\sin(2x)}{\tan x} & \text{for } 2 < x \leq 4, \end{cases}$$

b. Find  $\lim_{x \rightarrow \infty} \frac{5e^{2x} + 3e^x}{30x + e^{2x}}$ .

24. A rocket R is launched vertically from a pad P on the ground located 3 miles from the ground radar station S. Find the altitude and speed of the rocket at the moment when the rocket is 5 miles from the radar station, if the angle between SR and the horizontal is increasing at a rate of 12 radians per hour.



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25. A rectangular storage container with an open top and a square base is to have a volume of  $800 \text{ m}^3$ . The material for the base costs \$10 per square meter. The material for the sides costs \$2 per square meter. Find the dimensions for the cheapest such container.

**Do not write below this line.**

Question 1	U	S	E
Question 2	U	S	E
Question 3	U	S	E
Question 4	U	S	E
Question 5	U	S	E

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Page 13 is intentionally left blank, in case you need extra paper to show your work.

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Limit results:

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1, \quad \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = 0, \text{ and } \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1$$

Volumes of solids: