NAME: _____ **ALPHA**_____

NO CALCULATORS ALLOWED ON THIS PART SHOW ALL WORK IN THIS EXAM PACKAGE

PART II: LONG ANSWER. NO CALCULATORS. You must turn in Parts I and II, and the bubble sheet before using your calculator. Parts I,II and the bubble sheet are due **20 min prior to the end of the exam period.**

18. Let $f(x) = 4\ln(x+3)$.

a. State the domain of f and the equation of its vertical asymptote.

$x > -3$ or $(-3, \infty)$

$x = -3$ vertical asymptote

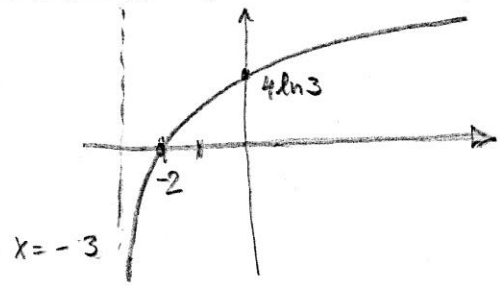
b. Find the x and y intercepts for the graph of f.

$x = ?$ $y = 0$ $4\ln(x+3) = 0$ | $y = ?$ $x = 0$ $y = 4\ln 3$.

$x + 3 = 1$

$\boxed{x = -2}$

c. Sketch the graph of f; incorporate the results from a,b in your graph.



d. Find the inverse function of f.

$y = 4\ln(x+3)$

$x = 4\ln(y+3)$

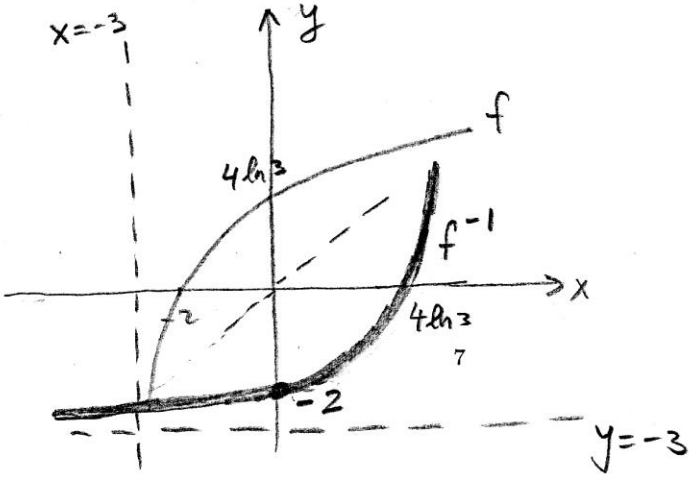
$\frac{x}{4} = \ln(y+3)$

$e^{x/4} = y + 3$

$y = \boxed{f^{-1}(x) = e^{x/4} - 3}$

1. D
2. D
3. B
4. B
5. A
6. C
7. A
8. D
9. C
10. A
11. B
12. A
13. A
14. E
15. B
16. A
17. D

e. On a new coordinate system, graph both f and f^{-1}; make sure to label at least one intercept and one asymptote for f^{-1}.



NAME: _____ ALPHA_____

NO CALCULATORS ALLOWED ON THIS PART SHOW ALL WORK IN THIS EXAM PACKAGE

19. a) State the limit definition of the derivative.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

b) Use the definition of the derivative to show that $\frac{d}{dx}(2x^2+x) = 4x+1$.

$$\lim_{h\to 0} \frac{2(x+h)^2+(x+h)-(2x^2+x)}{h} =$$

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2)+\cancel{x}+h-2x^2-\cancel{x}}{h} =$$

$$\lim_{h\to 0} \frac{\cancel{2x^2}+4xh+2h^2+h-\cancel{2x^2}}{h} = \lim_{h\to 0} \frac{h(4x+2h+1)}{h}$$

$$\lim_{h\to 0} (4x+2h+1) = 4x+1.$$

20. State and prove ONE of the following: Option 1: Product rule for computing the derivative of $f(x)g(x)$. Option 2: The derivative of $y = \sin x$ is $\frac{dy}{dx} = \ldots$.

Note: You may benefit from looking up the results listed on page 14.

Option 1

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$\lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} =$$

$$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x+h)g(x) + f(x+h)g(x) - f(x)g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)[g(x+h)-g(x)]}{h} + \lim_{h\to 0} \frac{g(x)[f(x+h)-f(x)]}{h}$$

$$= \lim_{h\to 0} f(x+h) \cdot \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} + g(x) \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= f(x)g'(x) + g(x)f'(x)$$ Done

Option 2

$$y = \sin x \qquad \frac{dy}{dx} = \cos x$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cos h - 1)}{h} + \lim_{h\to 0} \frac{\sin h \cos x}{h}$$

$$= \sin x \underbrace{\lim_{h\to 0} \frac{\cos h - 1}{h}}_{0} + \cos x \underbrace{\lim_{h\to 0} \frac{\sin h}{h}}_{1}$$

$$= (1)\cos x = \cos x$$ Done

NAME: _____ **ALPHA**_____

NO CALCULATORS ALLOWED ON THIS PART SHOW ALL WORK IN THIS EXAM PACKAGE

21. Consider the function $f(x) = \sqrt[3]{x+8}$.

a. Find the equation for the tangent line to $y = f(x)$ at $a = 0$.

$f(0) = \sqrt[3]{8} = 2$ Use $f(x) = (x+8)^{1/3}$

$f'(x) = \frac{1}{3}(x+8)^{-2/3} = \frac{1}{3}\frac{1}{\sqrt[3]{x+8}^{\,2}}$; $f'(0) = \frac{1}{3\sqrt[3]{8}^{\,2}} = \frac{1}{12}$

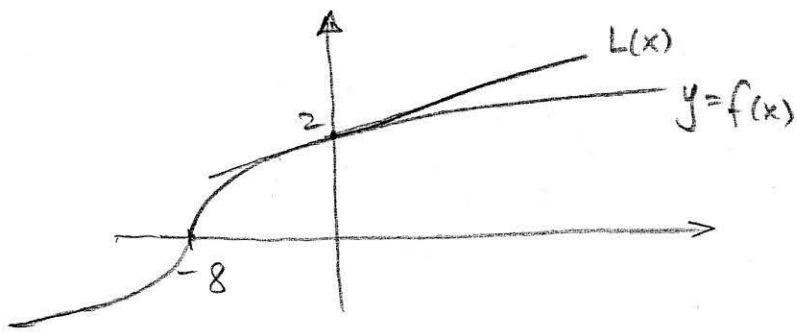
$\boxed{y = 2 + \frac{1}{12}x}$ $L(x) = 2 + \frac{x}{12}$

b. Use your linear approximation from a. to estimate $\sqrt[3]{8.24}$.

$x = 0.24$ $f(0.24) = \sqrt[3]{8.24}$

$\boxed{y \simeq 2 + \frac{1}{12}\,0.24} = 2 + \frac{24}{12 \cdot 100} = 2.02$

c. Sketch the graph of f (mark the intercepts on the graph), and the graph of the tangent line from a.



NAME: _____ ALPHA_____

YOU SHOULD TURN IN PARTS I, II, and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

PART III: LONG ANSWER. Calculators allowed AFTER TURNING IN PARTS I, II and the SCANTRON FORM. Parts I, II and the bubble sheet are due **20 min prior to the end of the exam period.**

22. Consider the function $f(x) = x^2 + \frac{25}{x^2+1}$

a. Use the first derivative test to identify the intervals where f is increasing or decreasing. If you use the calculator to assist your computations, make sure to show the relevant results.

$$f'(x) = 2x - \frac{50x}{(x^2+1)^2} = \frac{2x(x-2)(x+2)(x^2+6)}{(x^2+1)^2}$$

$- - -$ $++$ $- - -$ $++$ $f'(x)$

-2 0 2 x

$2x = \frac{50x}{(x^2+1)^2}$ $\quad x^2+1 = \pm 5$ $\quad f'(-3) = -9/2$

$(x^2+1)^2 = 25$ $\quad x^2+1 = 5$ $\quad f'(-1) = \frac{21}{2}$

$\boxed{x = \pm 2, \ x = 0}$ $\quad f'(1) = -21/2$

f increasing on $(-2, 0)$ and $(2, \infty)$

f decreasing on $(-\infty, -2)$ and $(0, 2$

b. Find the x and y coordinates of the local minima and local maxima for f

local minima

$x = -2 \quad f = 9$

$x = 2 \quad f = 9$

local max

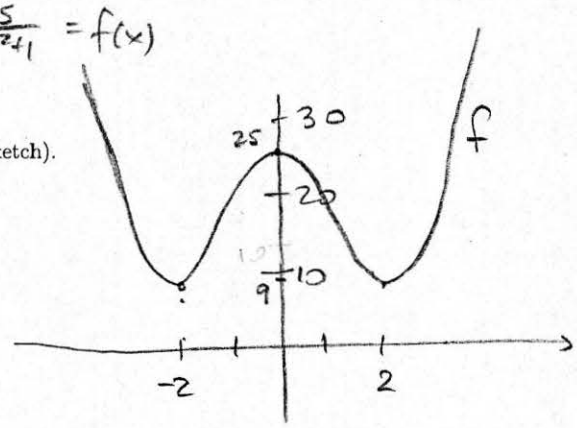
$x = 0 \quad f = 25$

c. Use the formula for f to determine whether f is even, odd, or neither.

$$f(-x) = (-x)^2 + \frac{25}{(-x)^2+1} = x^2 + \frac{25}{x^2+1} = f(x)$$

f is even

d. Sketch f (incorporate your results from a,b,c into your sketch).



e. Find the most general antiderivative for f.

$$\int \left(x^2 + \frac{25}{x^2+1}\right) dx = \frac{1}{3}x^3 + 25 \arctan x + C$$

NAME: _____ ALPHA_____

YOU SHOULD TURN IN PARTS I, II, and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

23. a. Find (show all work) $\lim_{x\to\pi} f(x)$, where

$$f(x) = \begin{cases} \frac{\sin(x)}{1+\cos x} & \text{for } 0 < x \le 2, \\ \frac{\sin(2x)}{\tan x} & \text{for } 2 < x \le 4, \end{cases}$$

$\pi \simeq 3.14$

Note: Use Precalc: $\frac{\sin(2x)}{\tan x} = \frac{2\sin x\cos x}{\frac{\sin x}{\cos x}} = 2\cos$

$\lim_{x\to\pi} \frac{\sin(2x)}{\tan x}$; $\frac{0}{0}$ $\lim_{x\to\pi} \frac{2\cos(2x)}{\sec^2(x)} = \frac{2(1)}{(1)} = 2$; $\lim_{x\to\pi} 2\cos^2 x = 2.$

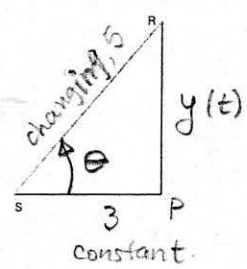
b. Find $\lim_{x\to\infty} \frac{5e^{2x}+3e^x}{30x+e^{2x}}$. $\frac{\infty}{\infty}$

1. Precalc: $\lim_{x\to\infty} \frac{e^{2x}\left[5+3e^{-x}\right]}{e^{2x}\left[1+\frac{30x}{e^{2x}}\right]} = \frac{5}{1}$.

2. L'H $\lim_{x\to\infty} \frac{10e^{2x}+3e^x}{30+2e^{2x}}$; $\frac{\infty}{\infty}$; Use L'H $\lim_{x\to\infty} \frac{20e^{2x}+3e^x}{4e^{2x}} = \lim_{x\to\infty}\left(5+\frac{3}{4}e^{-x}\right) =$

24. A rocket R is launched vertically from a pad P on the ground located 3 miles from the ground radar station S. Find the altitude and speed of the rocket at the moment when the rocket is 5 miles from the radar station, if the angle between SR and the horizontal is increasing at a rate of 12 radians per hour.

let $y = y(t)$ = the altitude, in miles.
t = time, in hours
$\theta = \theta(t)$ angle $\angle RSP$, in radians



$y^2 + 3^2 = 5^2$, $\boxed{y = 4 \text{ at that moment (miles)}}$

$\tan\theta(t) = \frac{y(t)}{3}$, $3\tan\theta(t) = y(t)$

$3 \cdot \frac{1}{\cos^2\theta} \frac{d\theta}{dt} = \frac{dy}{dt}$ Use $\cos\theta = \frac{3}{5}$

$3 \cdot \frac{1}{\frac{9}{25}} \cdot 12 = \frac{dy}{dt}$ $\frac{dy}{dt} = \frac{36 \cdot 25}{9} = \boxed{100 \text{ miles per hour} = \frac{dy}{dt}}$ speed.

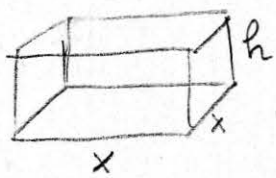
NAME: _____ Key ALPHA_____

YOU SHOULD TURN IN PARTS I, II, and the SCANTRON FORM PRIOR TO USING THE CALCULATOR ON THIS PART Show details of your work and box your answers.

25. A rectangular storage container with an open top and a square base is to have a volume of 800 m^3. Material for the base costs \$10 per square meter. Material for the sides costs \$2 per square meter. Find the dimensions for the cheapest such container.

let x = the side length of the base (in meters)

h = the height of the box, (meters)

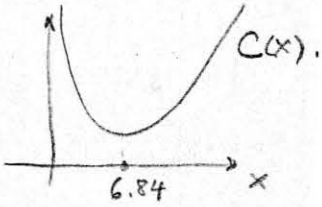


$V = x^2 h = 800$ (constraint)

$\text{Cost} = 10x^2 + 2(4xh) = 10x^2 + 8xh$ Use $h = \frac{800}{x^2}$

$C = 10x^2 + \frac{6400x}{x^2} = \boxed{10x^2 + 6400x^{-1}}$ Want min, $x > 0$

$\frac{dc}{dx} = 20x - 6400x^{-2} = 20x - \frac{6400}{x^2}$



$x \to 0$, $C \to \infty$ $x \to \infty$, $C \to \infty$

$20x^3 = 6400$

$x^3 = 320$

$x = \sqrt[3]{320} = \boxed{6.84}$ meters.

$x = 4\sqrt[3]{5}$

$h = \frac{800}{x^2} = 10\sqrt[3]{5}$

$\boxed{h = 17.10}$ meters

Do not write below this line.

Question 1	U	S	E
Question 2	U	S	E
Question 3	U	S	E
Question 4	U	S	E
Question 5	U	S	E

Draws a picture, Introduces Variables (with units), Labels Pictu

Sets up the volume requirement

Sets up the cost (objective) using both sides

Sets up the cost as single variable function

Finds the critical point, reasons it's the right ans (gives all dimensions, with units)

U: does none, or does some of the work but very sloppy work

S: does most of the work, possibly with small errors, but concept OK