Name:_____ Alpha:_____ Section:_____

This exam consists of 30 questions - 20 multiple choice and 10 free response. Do not discuss the contents of this exam with anyone until after 14 May 2010. Be sure to write your **name**, **alpha code**, and **section number** on this sheet and on any other sheets of paper you submit.

Part A – Hand Computation (25%). The first 3 problems must be worked out **without a calculator**. Show your work. Upon completing this section, turn it in to your instructor to receive the remainder of the exam. The maximum time recommended for this section is 30 minutes.

1. Evaluate the following:

a) $\dfrac{d}{dt}\left(\cos^2\left(2t-\pi\right)\right)$

b) $\dfrac{d}{dx}\left(\ln\left(\dfrac{2x}{3x-2}\right)\right)$

c) $\dfrac{d}{dt}\left(4t^2\sin\left(2t\right)\right)$

d) $\dfrac{d}{dx}\left(\tan^{-1}\sqrt{2x}\right)$

e) $\displaystyle\int_0^1\left(\cos\left(x\right)+3x^2+1\right)dx$.

2. Evaluate the following limits using any appropriate technique.

a) $\displaystyle\lim_{x\to\infty}\dfrac{2e^x}{x^2}$

b) $\displaystyle\lim_{x\to 0}\dfrac{x^3+2x+3}{3e^x}$

c) $\displaystyle\lim_{x\to\infty}\dfrac{x^3+2x+4}{3x^3-7x}$

3. For the curve with equation $f(x) = \dfrac{2}{4 - x^2}$:

    a) Give the equations for any vertical asymptotes; if none exist state why.

    b) Give the equations for any horizontal asymptotes; if none exist state why.

    c) Find all local maximums and local minimums; justify your classifications with an appropriate test.

    d) Using your information from parts a) through c), sketch and label the graph of $f(x)$.
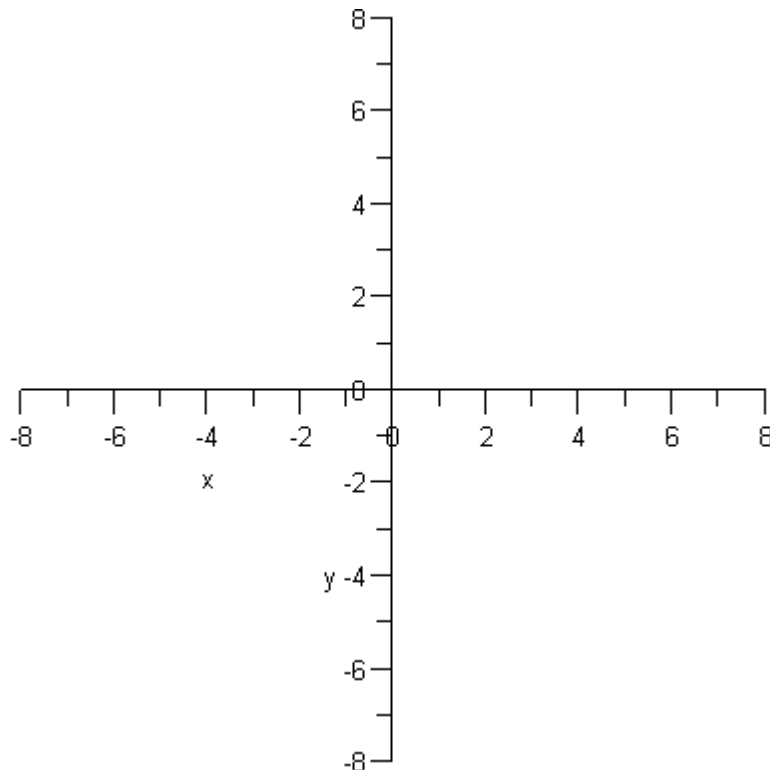
Name:_____   Alpha:_____   Section:_____

Part B – Multiple Choice (40%). These 20 problems are multiple choice. PUT YOUR ANSWERS ON THE SCANTRON SHEET. YOUR INSTRUCTOR MAY ALSO REQUIRE YOU TO CIRCLE YOUR ANSWERS ON THIS FORM.

1. The graph of $y^2 - \dfrac{x^2}{9} = 1$ is best described as:

a) a hyperbola   b) a parabola   c) a circle   d) an ellipse   e) something else

2. If g(x) = tan(x), which equation shifts g(x) $\pi/3$ units to the right, stretches it vertically by a factor of 3, and compresses it horizontally by a factor of 2

    a) $h(x) = 2\tan(3x - \dfrac{\pi}{3})$ b) $h(x) = 3\tan(2x - \dfrac{\pi}{3})$ c) $h(x) = 3\tan(2x + \dfrac{\pi}{3})$

    d) $h(x) = 3\tan(\dfrac{1}{2}x + \dfrac{\pi}{3})$ e) $h(x) = 3\tan(\dfrac{1}{2}x - \dfrac{\pi}{3})$

3. If $\lim\limits_{x \to \infty} f(x) = 8$, then

a) $f(x)$ is undefined for large values of x   b) $f(x)$ has no horizontal asymptotes

c) $f(x)$ has a vertical asymptote at $y = 8$ d) $f(x)$ has a horizontal asymptote at $y = 8$

e) $f(x)$ passes through the point $(\infty, 8)$

4. Which of the following is/ are true of one-to-one functions? They

a) pass the vertical line test   b) pass the horizontal line test   c) have inverses

    d) a, b, and c are all true   e) only b and c are true

5. Using the graph to the right, $\lim\limits_{x \to 2^+} f(x) =$

    a) does not exist
    b) 2
    c) 4
    d) 0
    e) -2
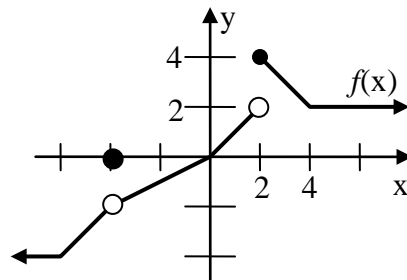
6. The radius of the circle $x^2 + y^2 + 6y + 2 = 0$ is:

    a) 2   b) $\sqrt{2}$ c) 7   d) $\sqrt{7}$ e) $\sqrt{11}$

7. The expression $3e^{(5x+2)}$ can be rewritten as:

a) $(5x+2)(\ln 3e)$ b) $3\left[e^{5x} + e^2\right]$ c) $3e^{5x}e^2$ d) $\ln 3 - \ln e^{(5x+2)}$ e) $(5x+2)(3e)$

8. If $f(x) = \dfrac{4x-1}{2x+3}$ then $f^{-1}(x)$, the inverse of f, is:

a) $\dfrac{2x+3}{4x-1}$
b) $\dfrac{1+3x}{4-2x}$
c) $\dfrac{3x+1}{2x-4}$
d) $\dfrac{3+2x}{x-4}$
e) $\dfrac{2x+3}{1-4x}$

Use the following information for questions 9 – 11:

$$f(x) = v(x)w(x)$$

x	v(x)	w(x)	v'(x)	w'(x)
1	3	2	4	6
2	1	3	5	7
3	7	2	7	9

$$g(x) = \dfrac{v(x)}{w(x)}$$

$$h(x) = v \circ w = v\big(w(x)\big)$$

9. g'(1) =

a) 0
b) $\dfrac{2}{5}$
c) $\dfrac{5}{2}$
d) $\dfrac{-5}{2}$
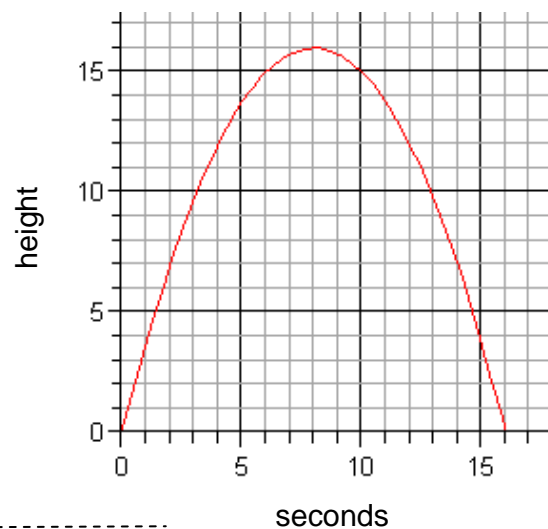e) cannot determine from given

information

---

10. h'(2) =

a) 49
b) 7
c) 63
d) 35
e) 12

---

11. $f'(2) =$

a) 11
b) 22
c) 33
d) 35
e) 52

Questions 12 and 13 refer to the diagram at the right, showing the position of an object over time :

12. What is the average velocity of the object

from t = 2 to t = 6 seconds?

a) 0 ft/s

b) 1 ft/s

c) 2 ft/s

d) 4 ft/s

e) 8 ft/s

---

13. What is the instantaneous velocity of the object at time t = 8 seconds?

a) – 2 ft/s
b) 0 ft/s
c) 2 ft/s
d) 16 ft/s
e) not enough information given

---

14. The linearization (linear approximation) of $f(x) = 2\ln(x)$ at $a = 1$ is given by

a) $L(x) = 2x - 2$
b) $L(x) = 2x$
c) $L(x) = 2x + 2$
d) $L(x) = \dfrac{2}{x}(x-1)$
e) $L(x) = 0$

15. If Newton's method is used to solve $x^3 - 3x^2 + 2x + 4 = 0$ with first approximation $x_1 = -1$, what is the second approximation, x_2?

    a) $-9/11$ b) $13/11$ c) $9/2$ d) 0   e) $-9/2$

---

16. For some number a and some function f, if $f'(a) = 0$ and $f''(x) < 0$, we can state that the function f has:

        a) a local maximum at x = a and is concave down at x = a

        b) a local maximum at x = a and is concave up at x = a

        c) a local minimum at x = a and is concave down at x = a

        d) a local minimum at x = a and is concave up at x = a

        e) a point of inflection at x = a

---

17. Using the graph to the right, the function f (x) is <u>not differentiable</u> at:

    a) x = 6

    b) x = 16

    c) x = 6 and x = 16

    d) $\{x \mid x = -3, 6, 16\}$

    e) f is differentiable for all x on interval (-5 , 17)

---
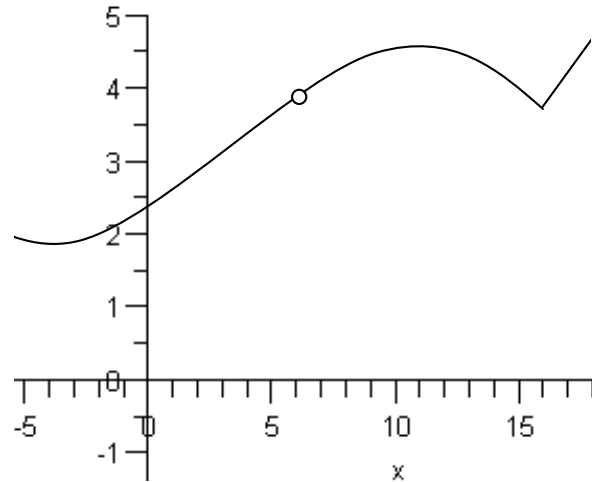
18. Suppose you want to verify that a root exists for a continuous function $f(x)$ on a closed interval, and you want to approximate the root. Which best describes the process to use?

    a) First Derivative Test to show a root exists; Second Derivative Test to approximate the root.
    b) Intermediate Value Theorem to show a root exists; Newton's Method to approximate the root.
    c) Mean Value Theorem to show the root exists ;Newton's Method to approximate the root.
    d First Derivative Test to show a root exists; Intermediate Value Theorem to approximate the root.
    e) Intermediate Value Theorem to show a root exists; Increasing/Decreasing Test to approximate the root.

---

19. Find $\dfrac{d}{dx}\left(\int_0^x t^2 \sin(t)\, dt\right)$.

    a) $\cos(x^2)$ b) $x^2 \sin(x)$ c) $\dfrac{x^3}{3}\sin(x)$ d) $\dfrac{x^3}{3}\cos(x)$ e) $-\dfrac{x^3}{3}\cos(x)$

20. The approximate area under the curve $f(x) = 12 - x^2$ for $0 \le x \le 3$ using R_3 (right endpoints, 3 rectangles) is:

          a) 31, which is an overestimate   b) 31, which is an underestimate

c) 22, which is an underestimate   d) 22, which is an overestimate   e) 26.5 exactly

Part C – Free Response (35%). Answer the last 7 problems fully on this examination form. Show the details of your work and clearly indicate your answers. Make sure your name and alpha code are on everything that you hand in.

1. For the function $f(x) = 3x^2 + 2x - 1$

    a. Use the limit definition of $f'(x)$ to prove that $f'(x) = 6x + 2$.

    b. Find the equation of the tangent line to $f(x)$ at the point Q (1, 4)

    c. Is f increasing or decreasing at point Q, and why?

2. Suppose a particular type of tree grows three feet higher each year and the radius of its trunk increases .1 foot per year. If the usable volume of wood in the main trunk is modeled as a cylinder ($volume = \pi r^2 h$), what is the rate of change of that volume per year when the radius is 0.2 feet and the trunk is 10 feet tall?

3. A particle moves along a straight line with velocity given by $v(t) = 3t^2 + t + 1$. The position at time t = 2 is given by $s(2) = 1$. Find a formula for the position function $s(t)$.

4. Find the absolute maximum and absolute minimum values of the function $g(x) = \left(x^2 - 1\right)^3$ on the interval [-1,2] and state the x values of where they occur.

5. An open top rectangular box is to be made from the least amount of cardboard possible according to the following specifications: the length of the base is three times its width, and the total volume is 9 cubic feet. (a) What are the length, width, and height of the box?

6. Suppose the displacement (position) of a bug is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s is measured in centimeters (cm) and $0 < t < 5$.

   a) Find the bug's velocity at time t

   b) Find the time the bug is at rest (stopped).

   c) What is the change in displacement from t = 0 to t = 5 sec?

   d) What is the total distance the bug traveled from t = 0 to t = 5 sec?

7. Use implicit or logarithmic differentiation as appropriate to find y' for the following functions:

a) $3xy + x^2 y = 5 - 2x + y$

b) $y = 2x^{\sin x}$