NAME: ANSWER KEY **ALPHA NUMBER:** ____________
INSTRUCTOR: ____________ **SECTION:** ____________

CALCULUS II (SM122, SM122A) — **FINAL EXAMINATION**

1330-1630 Monday 4 May 2009 — SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor and section on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS PERMITTED FOR THIS SECTION.

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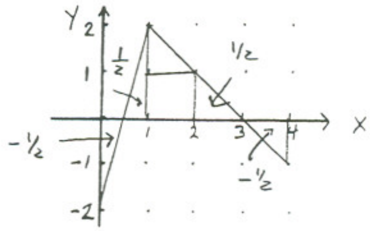
1. Determine $\int_0^4 f(x)dx$ for the function f whose graph on the right consists of 2 line segments.



a) -1 b) 1 c) 1.5
d) 2.5 e) 4.5

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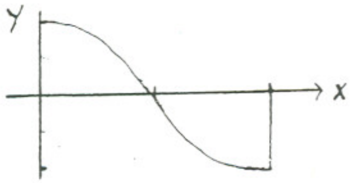
2. Determine the *total* area bounded between the curve $y = \cos(x)$ and the x - axis ($y = 0$) over the interval $[0, \pi]$.

$$A = 2\int_0^{\pi/2} \cos(x)\,dx = 2 \sin(x)\Big|_0^{\pi/2} = 2\sin(\pi/2) = 2$$



a) -1 b) 0
c) 1 d) 2
e) 2π

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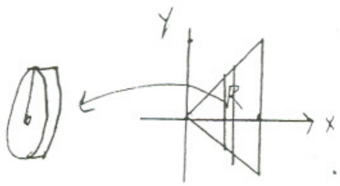
3. Find the volume of revolution obtained by rotating about the x – axis the region R bounded by $y = x$, $y = 0$, and $x = 1$.

$$V = \int_0^1 \pi r^2\,dx = \int_0^1 \pi x^2\,dx = \pi \frac{x^3}{3}\Big|_0^1 = \pi/3$$



a) 1/3 b) $\sqrt{2}$
c) 2 d) $\pi/3$
e) $\pi/2$

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4. Find $\int_1^\infty f(x)\,dx$ given that $\int_1^t f(x)\,dx = \frac{-2t}{t+1} + \frac{2}{e^t}$.

a) -2 b) ∞ c) 0
d) 2 e) does not exist

$$\int_1^\infty f(x)dx = \lim_{t\to\infty}\left[\int_1^t f(x)dx\right] = \lim_{t\to\infty}\left[\frac{-2t}{(t+1)} + \frac{2}{e^t}\right] = -2+0$$

5. After a u - substitution, the definite integral $\int_1^2 (3x^2+1)\sqrt{x^3+x}\,dx$ equals which of the following?

a) $\int_1^2 \sqrt{u}\,du$

b) $\int_2^{10} \sqrt{u}\,du$

c) $\int_1^2 \frac{2}{3}u^{3/2}\,du$

d) $\int_2^{10} \frac{2}{3}u^{3/2}\,du$

e) $\int_2^{10} \frac{1}{2}\frac{1}{\sqrt{u}}\,du$

$$\left[\begin{array}{l} \text{let } u = x^3 + x \Rightarrow du = (3x^2+1)dx \\ 1 \xrightarrow{x} 2 \Rightarrow 2 \xrightarrow{u} 10 \end{array}\right]$$

$$= \int_2^{10} \sqrt{u}\,du$$

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6. After one integration by parts, the indefinite integral $\int x^3 e^x dx$ could equal which of the following?

a) $x^3e^x - \frac{1}{3}\int x^2 e^x dx$

b) $x^3e^x - 3\int x^2 e^x dx$

c) $x^4e^x - 3\int x^2 e^x dx$

d) $x^3e^x - \int x^2 e^x dx$

e) none of the above

$$\left[\begin{array}{l} \text{let } u = x^3 \Rightarrow du = 3x^2 dx \\ \text{let } dv = e^x dx \Rightarrow v = e^x \end{array}\right]$$

$$= uv - \int v\,du$$

$$= x^3e^x - \int e^x 3x^2 dx$$

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7. Use two steps of Euler's method to approximate $y(2)$ if $y' = 2x + y$ and $y(0) = 1$. (Use step size h = 1.)

a) 0.1 b) 0.2 c) 2
d) 4 e) 6

x	y	y'	$\Delta y = y'h$	new y = old $y + \Delta y$	new x = old $x + h$
0	1	1	$1(1)=1$	$1+1=2$	$0+1=1$
1	2	$2(1)+2$	$4(1)=4$	$2+4=6$	$1+1=2$
2	6				

$\Rightarrow y(2) \doteq 6$

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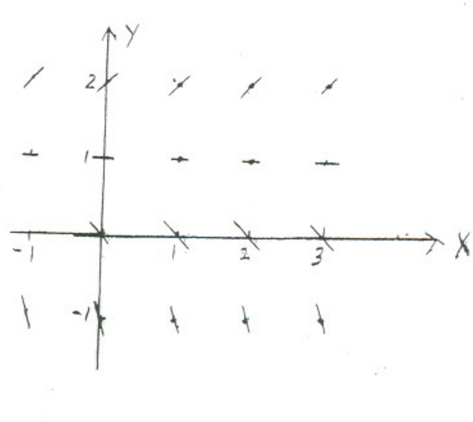
8. The direction field on the right could correspond to which one of the following differential equations?

a) $y' = x^2$ b) $y' = 1 - x$

c) $y' = x + y$ d) $y' = x^2 + y^2$

e) $y' = y - 1$

no change in slopes as x changes



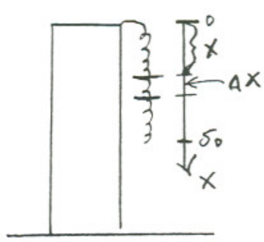
9. The function $y = xe^x$ is a solution to which one of the following differential equations?

a) $y' + y = e^x$ b) $y' - y = e^x$
c) $y' = -4y$ d) $y' - y = 0$
e) $y' = 16y$

$y = xe^x$
$y' = 1e^x + xe^x$
$\Rightarrow y' - y = e^x + xe^x - xe^x$
$= e^x$

10. A chain, 50 ft long, weighs 2 lbs/ft and hangs over the edge of a building 100 ft high. How much work is done in pulling the rope to the top of the building?

a) 10,000 ft-lbs
b) 2,500 ft-lbs
c) 1,000 ft-lbs
d) 500 ft-lbs
e) none of the above



ΔW = work to lift one segment to roof
$= f \cdot d =$ weight $\cdot\, x$
$= \left(2 \frac{lbs}{ft}\right)(\Delta x\ ft) \cdot x(ft)$
$\Rightarrow W = \int_0^{50} 2x\,dx$ ft-lbs
$= x^2\Big|_0^{50} = (50)^2 = 2{,}500$ ft-lbs

11. A simple series RC circuit consists of a capacitor with capacitance C = 0.5 farad, a resistor with resistance R = 3 ohms, and a constant EMF with E = 7 volts. The initial charge on the capacitor is Q = 4 coulombs.

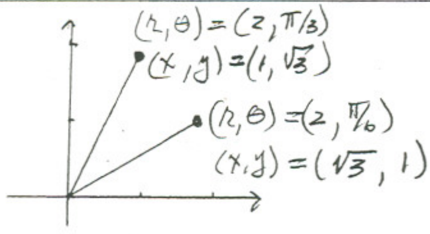
Kirchoff's law leads to which of the following differential equations for the charge $Q(t)$?

a) $2Q' + 3Q = 7$ b) $3Q' + 0.5Q = 7$
c) $3Q' + 2Q = 7$ d) $4Q' + 3Q = 7$
e) $7Q' + 2Q = 4$

$RI + \frac{1}{C}Q = E$, $I = Q'$
$\Rightarrow 3Q' + \frac{1}{.5}Q = 7$
$\Rightarrow 3Q' + 2Q = 7$

12. Find the best approximate distance between the two points in the plane whose polar coordinates (r, θ) are $(2, \pi/6)$ and $(2, \pi/3)$.
(Hint: first change to Cartesian coordinates.)

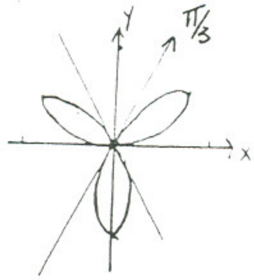
a) 0.6
b) 0.8
c) 1.0
d) 1.2
e) 1.4



$d = \sqrt{(1-\sqrt{3})^2 + (\sqrt{3}-1)^2}$
$= \sqrt{2(1-\sqrt{3})^2}$
$\doteq 1.04$

13. We learned that the area inside a polar region can be written as $A = \int_a^b \frac{1}{2} r^2 d\theta$. Find the area inside one loop of the 3-petal curve $r = \sin(3\theta)$. {Hint: Use your calculator or use the trig identity $\sin^2(z) = \frac{1-\cos(2z)}{2}$. }

a) π b) $\pi/3$
c) $\pi/6$ d) $\pi/12$
e) π^2



$$A = \int_0^{\pi/3} \frac{1}{2}\left[\sin^2(3\theta)\right] d\theta$$
$$= \int_0^{\pi/3} \frac{1}{2}\left[\frac{1-\cos(6\theta)}{2}\right] d\theta$$
$$= \frac{1}{4}\left[\theta - \frac{\sin(6\theta)}{6}\right]\Big|_0^{\pi/3}$$
$$= \frac{1}{4}\left[\left(\frac{\pi}{3}-0\right)-(0-0)\right] = \frac{\pi}{12}$$

14. The infinite harmonic series is given by $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots$.

The infinite alternating harmonic series is given by $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$.

Which of the following statements is true?

a) Both series converge to the same sum.
b) Both series converge, but to different sums.
c) Both series diverge.
d) The harmonic series converges, but the alternating harmonic series diverges.
e) The harmonic series diverges, but the alternating harmonic series converges.

15. Find the open interval of convergence for the power series centered at 10, $\sum_{n=1}^{\infty} \frac{(x-10)^n}{n!}$.

a) (5 , 15)
b) (-5 , 25)
c) (-20 , 40)
d) ($-\infty$, ∞)
e) none of the above

ratio test:

$$L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|x-10|^{\cancel{n+1}\,1}}{\cancel{(n+1)!}_{\,n+1}} \cdot \frac{\cancel{n!}}{\cancel{|x-10|^n}}$$

$$= \lim_{n\to\infty} \frac{|x-10|}{n+1} = 0 \text{ for all } x.$$

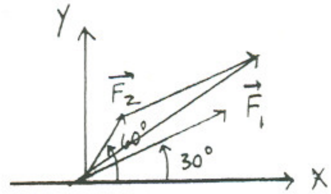
since $L = 0 < 1$ for all x $\Rightarrow$ interval of convergence $= (-\infty, \infty)$

16. Find the sum of the geometric series $16 + 12 + 9 + \ldots$

a) 32 b) 64 c) 40
d) 400 e) the series diverges

$= a + ar + ar^2 + \cdots$
where $a = 16$ and $r = 3/4$
$|r| < 1 \Rightarrow$ series converges to $\frac{a}{1-r}$
$$= \frac{16}{1-3/4} = 16 \cdot 4 = 64$$

17. Two forces $\vec{F}_1$ and $\vec{F}_2$ with magnitudes 4 and 2 Newtons respectively make angles of 30° and 60° respectively with the positive x-axis. If $\vec{F} = \vec{F}_1 + \vec{F}_2$, the angle $\vec{F}$ makes with the positive x – axis is closest to which of the following?



a) 35°
b) 40°
c) 45°
d) 50°
e) 55°

$\vec{F}_1 = \langle 4\cos(30^\circ), 4\sin(30^\circ)\rangle = \langle 2\sqrt{3}, 2\rangle$

$\vec{F}_2 = \langle 2\cos(60^\circ), 2\sin(60^\circ)\rangle = \langle 1, \sqrt{3}\rangle$

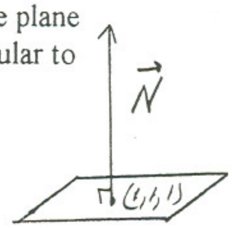
$\vec{F} = \vec{F}_1 + \vec{F}_2 = \langle 2\sqrt{3}+1, 2+\sqrt{3}\rangle$

$\theta = \tan^{-1}\left(\frac{2+\sqrt{3}}{2\sqrt{3}+1}\right) \doteq 39.9^\circ$

18. Which of the following expressions could result in a vector?

a) $\frac{\vec{c}\cdot\vec{a}}{\vec{a}\times\vec{b}}$ b) $\frac{\vec{c}}{\vec{a}\cdot\vec{b}}$ c) $(\vec{a}\cdot\vec{b})\times\vec{c}$ d) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$ e) none of these

19. Which of the following is an equation for the plane going through the point (1, 1, 1) and perpendicular to the line x = 3 + 2t, y = 1 + 3t, z = 5 + 4t?



$x = 3+2t$
$y = 1+3t$
$z = 5+4t$
$\vec{V} = \langle 2,3,4\rangle$

a) $2x + 3y + 4z = 9$
b) $3x + 1y + 5z = 9$
c) $1x + 1y + 1z = 9$
d) $2x + 3y + 4z = 0$
e) none of the above

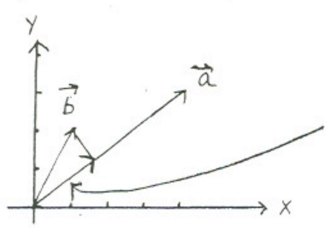
use $\vec{N} = \langle 2,3,4\rangle$

$\Rightarrow a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$\Rightarrow 2(x-1)+3(y-1)+4(z-1)=0 \Longrightarrow 2x+3y+4z=9$

20. Find the vector projection of $\vec{b} = \langle 1,2\rangle$ onto $\vec{a} = \langle 4,3\rangle$, $proj_{\vec{a}}\vec{b}$.

a) 4
b) 2
c) $\langle \frac{9}{5}, \frac{12}{5}\rangle$
d) $\langle \frac{8}{5}, \frac{6}{5}\rangle$
e) $\langle 8,6\rangle$



$proj_{\vec{a}}\vec{b} = \left(\vec{b}\cdot\frac{\vec{a}}{|a|}\right)\frac{\vec{a}}{|a|}$

$= \left(\langle 1,2\rangle\cdot\frac{\langle 4,3\rangle}{5}\right)\frac{\langle 4,3\rangle}{5}$

$= \left(\frac{4+4}{5}\right)\frac{\langle 4,3\rangle}{5} = 2\,\frac{\langle 4,3\rangle}{5}$

$= \langle \frac{8}{5}, \frac{6}{5}\rangle$

PART TWO. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES. **CALCULATORS PERMITTED FOR ALL PROBLEMS <u>EXCEPT</u> 27, 28, 29, 30.**

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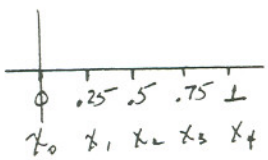
21. a) Use your calculator to approximate $\int_0^1 e^{x^2}\,dx$. (Show three decimal places in your answer.)

$\int(e\hat{}(x\hat{}2), x, 0, 1) = 1.463$

b) Simpson's rule using n subdivisions is

$$S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)].$$

Use Simpson's rule with 4 subdivisions, S_4, to approximate $\int_0^1 e^{x^2}\,dx$. (Show three decimal places in your answer.)



$$S_4 = \frac{.25}{3}\left[e^0 + 4e^{\frac{1}{16}} + 2e^{\frac{1}{4}} + 4e^{9/16} + e^{1}\right]$$

$$\doteq 1.464$$

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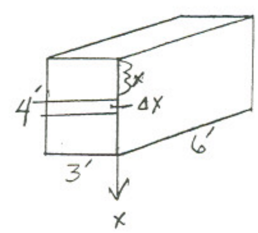
22. We learned that hydrostatic force = (pressure)x(area). An aquarium 6 ft long, 3 ft wide, and 4 ft deep is full of water. Use the fact that water pressure at a depth of d ft is $(62.5)(d)\,lbs/ft^2$ to find:

a) The hydrostatic force on the bottom of the aquarium.



$hf = (\text{pressure}) \times (\text{area})$

$= (62.5)(4)\left(\frac{lbs}{ft^2}\right)(3\times 6)\,ft^2 = 4{,}500\text{ lbs.}$

b) The hydrostatic force on one end (4' x 3') of the aquarium.

Δhf = hydro force against one strip

$= (\text{pres})(\text{area}) = (62.5x)\frac{lbs}{ft^2}(3\Delta x)\,ft^2$

$\Rightarrow hf = \int_0^4 (62.5)(3)x\,dx$

$= 1{,}500\text{ lbs.}$

23. Write out the first three terms of each of the following series and tell whether each series converges or diverges (you must justify your answers):

a) $\sum_{n=1}^{\infty} \frac{3n}{4n+1} = \frac{3}{5} + \frac{6}{9} + \frac{9}{13} + \cdots$

$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{3n}{4n+1} = \lim_{n\to\infty} \frac{(3n)/n}{(4n+1)/n} = \lim_{n\to\infty} \frac{3}{4+1/n} = \frac{3}{4}$

Since $\lim_{n\to\infty} |a_n| \neq 0 \Rightarrow$ the series diverges by the divergence test.

b) $\sum_{n=1}^{\infty} \frac{n^2}{3^n} = \frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \cdots$

By the ratio test $L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{(n+1)^2}{\cancel{3^{n+1}}_{3}} \cdot \frac{\cancel{3^n}}{(n)^2}$

$= \lim_{n\to\infty} \frac{(n+1)^2}{n^2} \cdot \frac{1}{3} = \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot \frac{1}{3} = \lim_{n\to\infty} (1+\underbrace{1/n}_{})^2 \cdot \frac{1}{3} = \frac{1}{3}$

1

Since $L = \frac{1}{3} < 1 \Rightarrow$ the series converges.

24. Assume that $y(t)$ (lbs) is the amount of radio-active material at time t (hours).

If $y(t)$ satisfies the differential equation $\frac{dy}{dt} = -.069y$, and $y(0) = 10$,

solve for y(t) and determine approximately the half-life (hours) for this radio-active material.

$\frac{dy}{dt} = -.069y \xrightarrow[\text{vars}]{\text{sep}} \frac{1}{y}\,dy = -.069\,dt \Rightarrow \int \frac{1}{y}\,dy = \int -.069\,dt \Rightarrow$

$\ln|y| = -.069t + c \Rightarrow e^{\ln|y|} = e^{(-.069t+c)} \Rightarrow \ln|y| = e^c e^{-.069t}$

$\Rightarrow y = Ae^{-.069t}$. $y(0) = 10 \Rightarrow 10 = Ae^0 \Rightarrow A = 10$

$\Rightarrow \boxed{y = 10e^{-.069t}}$

for half life, find t when $y = 5 \Rightarrow 5 = 10e^{-.069t}$

$\Rightarrow \frac{1}{2} = e^{-.069t} \Rightarrow \ln(\frac{1}{2}) = -.069t$

$\Rightarrow t \doteq \frac{-.69}{-.069} = \boxed{10 \text{ hrs}}$

25. Find the first 4 non-zero terms of the Maclaurin series for $f(x) = (1 + x)^{-2}$ by finding the Maclaurin series coefficients directly.

$f(x) = (1+x)^{-2} \Rightarrow f(0) = 1$

$f'(x) = -2(1+x)^{-3} \Rightarrow f'(0) = -2$

$f''(x) = 3\cdot 2(1+x)^{-4} \Rightarrow f''(0) = 3\cdot 2$

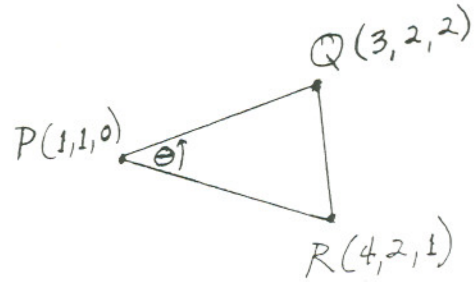
$f^{IV}(x) = -4\cdot 3\cdot 2(1+x)^{-5} \Rightarrow f'''(0) = -4\cdot 3\cdot 2$

$f(x) = f(0) + \frac{f'(0)x}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots$

$= 1 - 2x + \frac{3\cdot\cancel{2}x^2}{\cancel{2}} - \frac{4\cdot\cancel{3}\cdot\cancel{2}x^3}{\cancel{3}\cdot\cancel{2}} + \cdots$

$= 1 - 2x + 3x^2 - 4x^3 + \cdots$

26. For the triangle shown on the right whose vertices are the points **P**(1, 1, 0), Q(3, 2, 2), and R(4, 2, 1):



a) Find the vectors $\overrightarrow{PQ}$, $\overrightarrow{PR}$ and the interior angle θ.

$\overrightarrow{PQ} = \langle 2, 1, 2 \rangle$, $\overrightarrow{PR} = \langle 3, 1, 1 \rangle$

$\theta = \cos^{-1}\left(\frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}\right) = \cos^{-1}\left(\frac{6+1+2}{\sqrt{9}\sqrt{11}}\right)$

$\doteq 25.2^\circ$ or $.44$ rads

b) Find the area of the triangle $\Delta\, PQR$.

area $\Delta PQR = \frac{1}{2}\left|\overrightarrow{PR} \times \overrightarrow{PQ}\right|$

$= \frac{1}{2}\sqrt{1+16+1}$

$= \frac{1}{2}\sqrt{18} = \frac{3}{2}\sqrt{2}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = \langle 1, -4, 1 \rangle$

c) Find an equation for the plane going through the three points P, Q, and R.

use $P(1,1,0)$ and $\overrightarrow{N} = \langle 1, -4, 1 \rangle$

$\Rightarrow 1(x-1) - 4(y-1) + 1(z-0) = 0$

$x - 4y + z = -3$

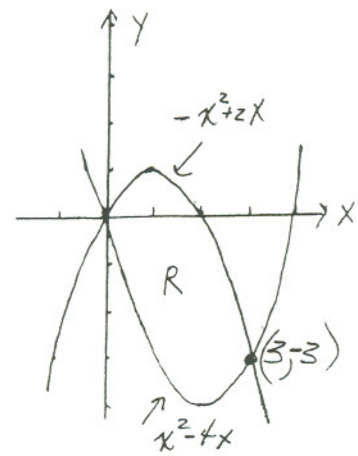
CALCULATORS NOT PERMITTED FOR 27, 28, 29, 30.

27. a) Evaluate $\int_0^1 (4x^3+8x+9)\,dx$.

$$= x^4+4x^2+9x\Big|_0^1 = 1+4+9 = 14$$

b) Find the area of the region R sketched on the right bounded by the two parabolas $y=x^2-4x$ and $y=-x^2+2x$.

pt of intersection: $x^2-4x=-x^2+2x$
$2x^2-6x=0$
$2x(x-3)=0$
$\Rightarrow x=0$ or 3.



$$\text{area} = \int_0^3 \left[(-x^2+2x)-(x^2-4x)\right]dx$$

$$= \int_0^3 \left[-2x^2+6x\right]dx$$

$$= -\frac{2}{3}x^3+3x^2\Big|_0^3 = -\frac{2}{3}(3)^3+3(3)^2 = -\frac{2}{\cancel{3}}(\cancel{27}^{9})+27$$

$$= 9$$

28. Evaluate the following indefinite integrals showing all of your steps:

a) $\int [\sin(3x)]^{10}\cos(3x)\,dx$

by substitution:
let $u=\sin(3x) \Rightarrow du = 3\cos(3x)\,dx$
$\Rightarrow \frac{1}{3}du = \cos(3x)\,dx$

$$= \int u^{10}\cdot\frac{1}{3}\,du$$

$$= \frac{1}{3}\,\frac{u^{11}}{11}+C = \frac{1}{33}\left[\sin(3x)\right]^{11}+C$$

b) $\int \frac{7t-11}{(t-1)(t-2)}\,dt$

by partial fractions:

$$\frac{7t-11}{(t-1)(t-2)} = \frac{A}{t-1}+\frac{B}{t-2} \Rightarrow 7t-11 = A(t-2)+B(t-1)$$

let $t=2$: $14-11 = 0+B(2-1)$
$\Rightarrow \boxed{B=3}$

let $t=1$: $7-11 = A(-1)$
$\Rightarrow \boxed{A=4}$

$$= \int\left(\frac{4}{t-1}+\frac{3}{t-2}\right)dt$$

$$= 4\ln|t-1|+3\ln|t-2|+C$$

$$= \ln\left|(t-1)^4(t-2)^3\right|+C$$

29. a) Evaluate $\int x e^{5x}\,dx$ showing all steps.

use integration by parts

let $u = x \Rightarrow du = 1\,dx$

let $dv = e^{5x}dx \Rightarrow v = \frac{e^{5x}}{5}$

$$= uv - \int v\,du$$

$$= \frac{xe^{5x}}{5} - \int \frac{e^{5x}}{5}\,dx$$

$$= \frac{xe^{5x}}{5} - \frac{e^{5x}}{25} + C$$

b) Use separation of variables to solve for y explicitly if $\dfrac{dy}{dx} = \dfrac{\cos(3x)}{y^2}$.

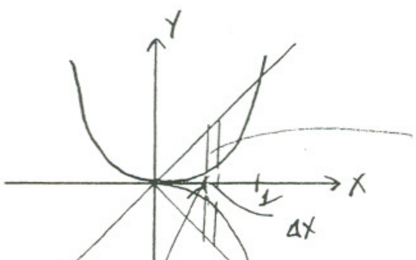
$$y^2\,dy = \cos(3x)\,dx$$

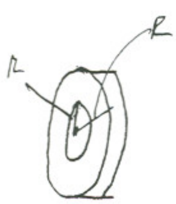
$$\Rightarrow \int y^2\,dy = \int \cos(3x)\,dx$$

$$\Rightarrow \frac{1}{3}y^3 = \frac{1}{3}\sin(3x) + C$$

$$\Rightarrow y^3 = \sin(3x) + 3C \Longrightarrow y = \sqrt[3]{\sin(3x) + A}$$

30. Sketch the region R bounded by the curves $y = x^2$ and $y = x$, and find the volume of the solid obtained by rotating the region R about the x – axis. (Use the washer method.)





$$\Delta V = \text{vol of 1 washer}$$
$$= \pi(R^2 - r^2)\Delta x$$
$$= \pi((x)^2 - (x^2)^2)\Delta x$$

$$V = \int_0^1 \pi(x^2 - x^4)\,dx$$

$$= \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \pi\left(\frac{5-3}{15}\right) = \frac{2}{15}\pi$$