NAME:___________________________________ ALPHA NUM:___________
INSTRUCTOR: ____________________________ SECTION: _______________

CALCULUS II (SM122, SM122A)   FINAL EXAMINATION

0755-1055 13 May 2010   SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor, and section number on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS ARE PERMITTED FOR THIS MULTIPLE CHOICE SECTION.

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1. A standard substitution converts the integral $\int x^2 e^{x^3+1} dx$ to which of the following?
a. $\int ue^u du$ b. $\int e^u du$ c. $\int \frac{1}{3} ue^u du$ d. $\int \frac{1}{3} e^u du$ e. $\int 3ue^u du$

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2. By the first part of the Fundamental Theorem of Calculus, the derivative of
$g(z) = \int_{\pi}^{z} \sqrt{1+\cos(t)}\, dt$ is
a. $\sin(z)\sqrt{1+\cos(z)}$
b. $\sqrt{1+\cos(z)}$
c. $-\sin(z)\sqrt{1+\cos(z)}$
d. $\sqrt{1-\sin(z)}$
e. $-\frac{1}{2}\sin(z)(1+\cos(z))^{-1/2}$

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3. Which of the following integrals gives the area of the region bounded by the curves $y = 2x^2$ and $y = x^2 + 4$?
a. $\int_0^2 (3x^2+4)dx$ b. $\int_0^2 (x^2-4)dx$ c. $\int_{-2}^2 (3x^2+4)dx$ d. $\int_{-2}^2 (x^2-4)dx$ e. $\int_{-2}^2 (4-x^2)dx$

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4. A spring's constant is $k = 6$ lb/ft. How much work (in ft-lbs) does it take to stretch it to 3 ft beyond its natural length?
a. 1/3   b. 1   c. 3   d. 9   e. 27

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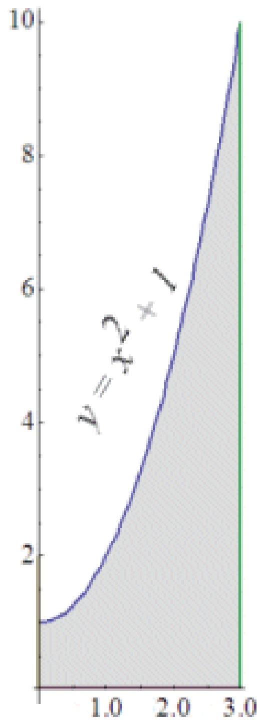
5. Which expression gives the volume of the solid formed by rotating the pictured shaded region (bounded by $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 3$) about the x-axis?



a. $\int_0^3 \pi(x^2 + 1)^2 dx$

b. $\int_0^1 \pi(y - 1) dy$

c. $\int_0^3 \pi(x^4 + 1) dx$

d. $\int_0^{10} \pi(y - 1)^2 dy$

e. $\int_0^1 \pi 3^2 dy + \int_1^{10} \pi\left(3^2 - (y - 1)\right) dx$

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6. The best u for evaluating the integral $\int_0^1 \frac{x}{e^{2x}} dx$ using integration by parts (writing the integral as $\int u\, dv$) is

a. $2x$ b. x c. e^{2x} d. e^{-2x} e. $\ln(2x)$

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7. A 20 foot sailboat's deck is measured across every 5 ft from bow to stern resulting in the following 5 measurements:

Distance from bow in ft	0	5	10	15	20
Width in ft	0	6	10	12	8

Using Simpson's rule with $n = 4$ to approximate the area of the deck gives, to the nearest square ft:

a. 140 b. 153 c. 160 d. 167 e. 180

8. For what value of c will the function f below be a probability density function?

$$f(x) = \begin{cases} \dfrac{c}{x^3}, & \text{if } x > 1 \\ 0, & \text{if } x \leq 1 \end{cases}$$

a. $1/4$ b. $1/2$ c. 1 d. 2 e. 4

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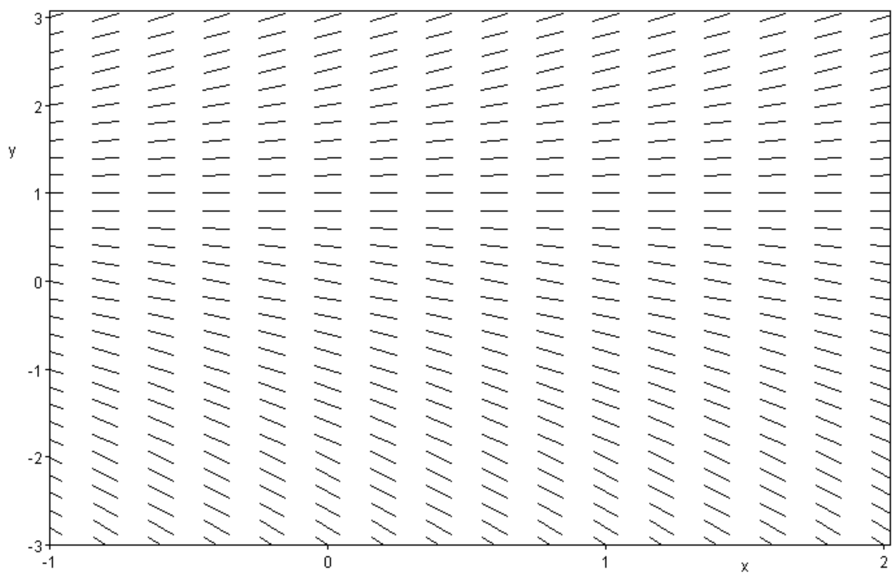
9. Given the direction field shown for a differential equation (note that the window view of the $xy-$ plane goes from -1 to 2 in the x direction, and from -3 to 3 in the y direction), if the initial condition is $y(0) = -1$ then which value below is closest to the value of the solution at $x = 1$?



a. -2 b. -1 c. 0 d. 1 e. 2

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10. Recall that the voltage drops across resistors, capacitors, and inductors are given by $E_R = RI$, $E_C = Q/C$, and $E_L = LI'$ respectively, and that the derivative of charge is current (i.e., $Q' = I$). What then is the differential equation describing current for a circuit consisting of a 4 henry inductor, an 8 ohm resistor, and a constant voltage of 16 volts?

a. $8I' + 4I = 16$
b. $4I' + 8I = 16$
c. $I'/4 + 8I = 16$
d. $8I' + I/4 = 16$
e. $I'/4 + I/8 = 16$

11. Five sets of polar coordinates (r, θ) are given. Four of them all represent the same point. Which set of polar coordinates represents a DIFFERENT point from the other four?

a. $\left(2, \frac{\pi}{4}\right)$ b. $\left(-2, -\frac{3\pi}{4}\right)$ c. $\left(2, \frac{9\pi}{4}\right)$ d. $\left(-2, \frac{5\pi}{4}\right)$ e. $\left(2, -\frac{5\pi}{4}\right)$

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12. The sequence with nth term $a_n = \frac{4n^2-3}{2n^2+1}$

a. converges to -3
b. converges to $1/2$
c. converges to 2
d. converges to 4
e. diverges

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13. Find the sum of the geometric series $\sum_{n=0}^{\infty} \frac{3^n}{4^{n+1}} = \frac{1}{4} + \frac{3}{4^2} + \frac{3^2}{4^3} + \cdots$

a. $\frac{1}{5}$
b. $\frac{3}{4}$
c. 1
d. $\frac{4}{3}$
e. 4

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14. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

a. 0 b. 1/3 c. 1 d. 3 e. ∞

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15. The Maclaurin series for e^{4x} starts as

a. $1 + 4x + 8x^2 + \frac{32}{3}x^3 + \cdots$
b. $e^{4x} + 4xe^{4x} + 8x^2e^{4x} + \cdots$
c. $1 + 4x + 16x^2 + 64x^3 + \cdots$
d. $1 + 4x + 8x^2 + \frac{64}{3}x^3 + \cdots$
e. $1 - 4x + 8x^2 - \frac{64}{3}x^3 + \cdots$

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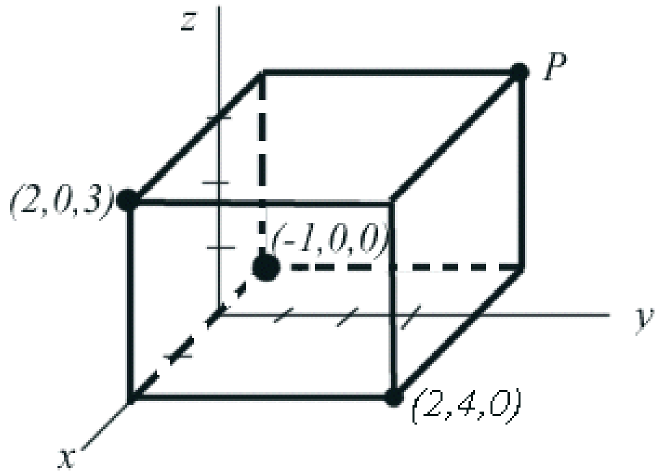
16. For the rectangular box as drawn and labeled, the coordinates of the corner P are

a. $(-1,4,3)$
b. $(-1,3,4)$
c. $(3,4,-1)$
d. $(4,3,-1)$
e. $(3,-1,4)$



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17. Given $\mathbf{a} = -\mathbf{i} + 4\mathbf{j} + \mathbf{k} = \langle -1,4,1 \rangle$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k} = \langle -2,1,-3 \rangle$ find the vector projection of **b** onto **a**, $\text{proj}_{\mathbf{a}}\mathbf{b}$.

a. $\langle -\frac{6}{14}, \frac{3}{14}, -\frac{9}{14} \rangle$
b. $\langle \frac{6}{14}, \frac{3}{14}, \frac{9}{14} \rangle$
c. $\langle \frac{6}{14}, -\frac{3}{14}, \frac{9}{14} \rangle$
d. $\langle \frac{1}{6}, \frac{4}{6}, \frac{1}{6} \rangle$
e. $\langle -\frac{1}{6}, \frac{4}{6}, \frac{1}{6} \rangle$

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18. For **i**, **j**, **k** the standard unit basis vectors, which of the following is a vector?

a. $\mathbf{i} \cdot \mathbf{j}$ b. $(\mathbf{i} \times \mathbf{j}) \cdot \mathbf{k}$ c. $|\mathbf{i} + 2\mathbf{k}|$ d. $\mathbf{j} \times \mathbf{k}$ e. $\mathbf{i} \cdot \mathbf{i} + \mathbf{j} \cdot \mathbf{k}$

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19. Which of the following is an equation of the plane determined by points $P(2,4,7)$, $Q(1,3,8)$, and $R(-3,3,7)$?

a. $x - 5y - 4z = -46$
b. $2x + 4y + 7z = 0$
c. $2x + y - 3z = 7$
d. $2x + 3y + z = 1$
e. $2x + y - z = 1$

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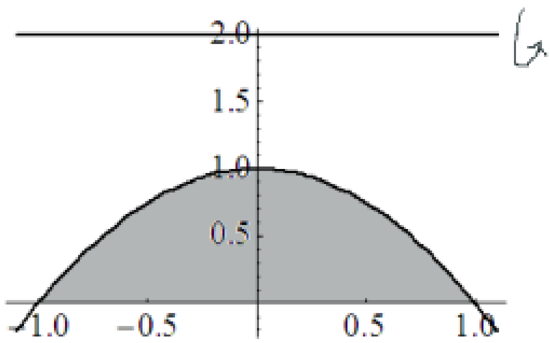
20. Parametric equations for the line through points $(2,3,1)$ and $(4,2,2)$ are

a. $x = 2 + 2t, \quad y = 3 - t, \quad z = 1 + t$
b. $x = -2 + 2t, \quad y = -3 - t, \quad z = -1 + t$
c. $x = 2 + 2t, \quad y = 1 - 3t, \quad z = 1 + t$
d. $x + 3y + z = 12$
e. $(x - 2) + (y - 3) + (z - 1) = 2$

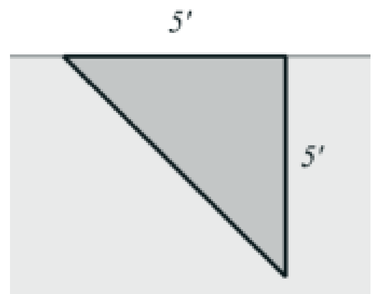
PART TWO: FREE RESPONSE (50%). The remaining problems are not multiple choice. Answer them on this test paper in the blank space provided. Show the details of your work and box your answers.

21. Let R be the region bounded by the $x-$ axis and the parabola $y = 1 - x^2$ as drawn. Find the volume of the solid generated when R is revolved about the horizontal line $y = 2$.



22. A chain that weights 3 lbs/ft is used to lift a 200 lb anchor 60 ft to the deck. (Assume the anchor is always above water; its weight is constant.) Find the total work done.

23. Find the hydrostatic force on one face of a plate as drawn – it is a vertical isosceles right triangle with shortest sides of length 5 ft and one of those short sides at the surface of the water. Assume that the weight of water is 62.5 lbs/ft^3.



24. Consider the differential equation $y' = -2y$ with initial condition $y(0) = 3$.
a. Use Euler's method with two steps of size $1/4$ to approximate $y(1/2)$.
b. Sketch the direction field of the differential equation, including at least one slope line in each quadrant.
c. Use the separable equation method to solve the equation with given initial condition and find $y(1)$ exactly, showing your work.

25. Sketch the curve with polar equation $r = 2 + \sin(\theta)$ and shade the region inside the curve but above the $x -$ axis. Find the area of the shaded region. (Hint: use your calculator to integrate.)

26. Consider the vector equation $\mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle$.

a. Sketch the curve it describes.

b. Find parametric equations for the tangent line to the curve at the point $(1, \pi/2, 0)$.

CALCULATORS ARE NOT PERMITTED FOR PROBLEMS 27, 28, 29, 30.

27. Evaluate:
a. $\int x\cos(3x)\,dx$

b. $\int \frac{x+25}{x^2-5x+4}\,dx$

28. Given $\mathbf{a} = -\mathbf{j} - 2\mathbf{k} = \langle 0, -1, -2 \rangle$ and $\mathbf{b} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k} = \langle 3,1,-4 \rangle$ find
a) $|\mathbf{a}|$
b) $3\mathbf{b} - 2\mathbf{a}$
c) $\mathbf{a} \cdot \mathbf{b}$
d) $\mathbf{a} \times \mathbf{b}$

29. Find the Taylor series for $f(x) = \ln(x)$ centered at $a = 2$. Write out the first 4 non-zero terms.

30. Prove the Mean Value Theorem for Integrals
"If f is continuous on $[a, b]$, then there is a number c in $[a, b]$ such that $f(c) = \frac{1}{b-a}\int_a^b f(t)\,dt$."
by applying the Mean Value Theorem for derivatives
"If f is continuous on $[a, b]$ and differentiable on (a, b), then there is a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$."
to the function $F(x) = \int_a^x f(t)\,dt$.