NAME:______________________________________ ALPHA NUMBER:__________
INSTRUCTOR: _______________________________ SECTION: ____________

1330-1630 06 May 2011 SHOW ALL WORK IN THIS TEST PACKAGE

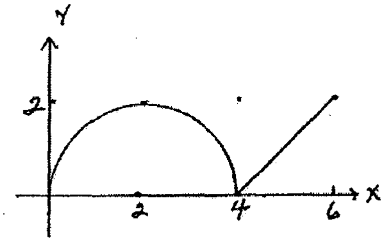
Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor, and section number on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS ARE PERMITTED FOR THIS MULTIPLE CHOICE SECTION.

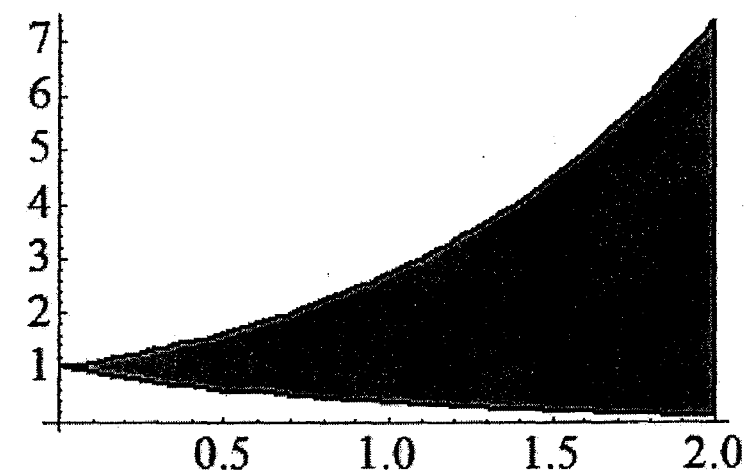
1. The graph of $y = f(x)$ on the right consists of a semicircle and a line segment.
$\int_0^6 f(x)dx$ is closest to

a) 0 b) 2 c) 4 d) 6 e) 8

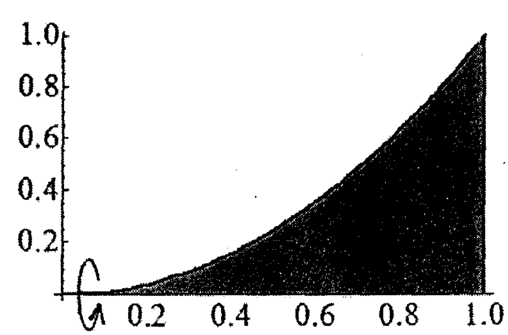


2. The area of the region bounded by $y = e^x, y = e^{-x}, x = 2$ is closest to

a) 5.5
b) 6.5
c) 7.5
d) 8.5
e) 9.5



3. Find the volume of the solid of revolution formed by rotating the region under the curve $y = x^2$ from $x = 0$ to $x = 1$ about the x-axis.
a) $\pi/5$
b) $\pi/4$
c) $\pi/3$
d) $\pi/2$
e) π



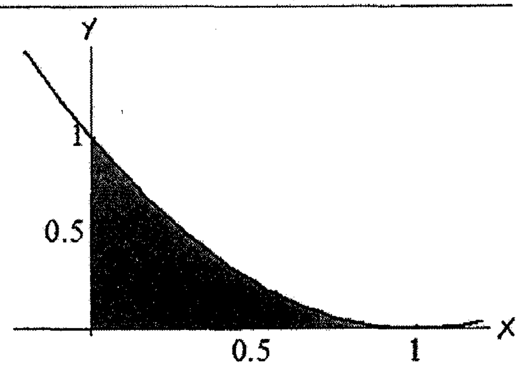
4. To evaluate the integral $\int \frac{dx}{\sqrt{x}(3+\sqrt{x})^2}$ the best substitution would be:

a) $u = \sqrt{x}$ b) $u = x$ c) $u = 1/\sqrt{x}$ d) $u = 3 + \sqrt{x}$ e) cannot be solved with substitution

5. Correctly applying the "integration by parts" procedure once to the anti-differentiation problem $\int x^2 \cos(5x)dx$ could produce the result:

a) $\int x^2 \cos(5x)dx = \frac{x^2}{5}\sin(5x) - \frac{2}{5}\int x \sin(5x)dx$
b) $\int x^2 \cos(5x)dx = \frac{x^2}{5}\sin(5x) + \frac{2}{5}\int x \sin(5x)dx$
c) $\int x^2 \cos(5x)dx = -\frac{x^2}{5}\sin(5x) - \frac{2}{5}\int x \sin(5x)dx$
d) $\int x^2 \cos(5x)dx = \frac{x^2}{5}\sin(5x) - \frac{2}{5}\int x \cos(5x)dx$
e) $\int x^2 \cos(5x)dx = -\frac{x^2}{5}\sin(5x) + \frac{2}{5}\int x \cos(5x)dx$

6. For the function f whose graph is pictured, consider approximating $\int_0^1 f(x)dx$ with 8 subintervals and left-hand sums (L_8), right-hand sums (R_8), and the trapezoidal rule (T_8). Which of these relationships is true? ("I" represents the exact value of the integral.)



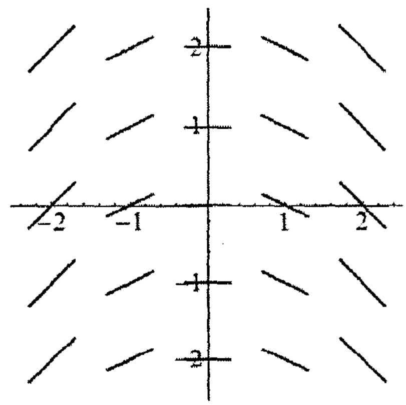
a) $L_8 < T_8 < I < R_8$
b) $L_8 < I < T_8 < R_8$
c) $R_8 < T_8 < I < L_8$
d) $R_8 < I < T_8 < L_8$
e) There is not enough information to determine the relationships

7. If $\int_0^\infty f(t)dt$ is ***divergent*** then $\lim_{T\to\infty} \int_0^T f(t)dt$ can be which of the following:

a) 0 b) e c) 10 d) ∞ e) any of these

8. For which value of r is $y = e^{rx}$ a solution to the differential equation $\frac{dy}{dx} + 4y = 0$?
a) - 4 b) 0 c) 4
d) 2 e) none of these

9. The direction field at right represents which of the following differential equations?



a) $\frac{dy}{dx} = -y$ b) $\frac{dy}{dx} = y$

c) $\frac{dy}{dx} = -x$ d) $\frac{dy}{dx} = x$

e) $\frac{dy}{dx} = -x^2$

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10. Apply Euler's method to the differential equation $\frac{dy}{dx} = 3x - y$ with initial condition $y(1) = 2$. Use a step size of $h = 0.5$ to estimate $y(2)$. The result is that $y(2)$ is approximately

a) 0
b) 2.5
c) 3.0
d) 3.5
e) 4.75

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11. Which of the following pairs of polar coordinates, (r, θ), does *NOT* correspond to the point with rectangular coordinates $(x, y) = \left(-\sqrt{2}, -\sqrt{2}\right)$?

a) $\left(2, -\frac{3\pi}{4}\right)$ b) $\left(-2, \frac{\pi}{4}\right)$ c) $\left(2, \frac{5\pi}{4}\right)$
d) $\left(-2, \frac{9\pi}{4}\right)$ e) $\left(-2, -\frac{3\pi}{4}\right)$

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12. We showed that the area of a polar region bounded by the curve $r = f(\theta)$ is given by $A = \int_a^b \frac{1}{2} r^2 d\theta$. The area enclosed by one loop of the four petal curve $r = \cos(2\theta)$ is given by:

a) $\int_0^{2\pi} \frac{1}{2}(\cos(2\theta))^2 d\theta$
b) $\int_0^{\pi/4} \frac{1}{2}(\cos(2\theta))^2 d\theta$
c) $\int_0^{\pi/4} (\cos(2\theta))^2 d\theta$
d) $\int_0^{\pi/2} (\cos(2\theta))^2 d\theta$
e) none of the above

13. What is the limit of the sequence given by formula $a_n = \frac{2-3n^2}{5+n^2}$?

a) $-\infty$ b) -3 c) $-3/5$
d) $2/5$ e) 2

14. The sum of the geometric series $\frac{3}{7} + \frac{(3)(5)}{7^2} + \frac{(3)(5^2)}{7^3} + \cdots$ is

a) $3/5$ b) $7/5$
c) $3/2$ d) 5
e) There is no sum; the series diverges

15. The radius of convergence for the power series $\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{3^n}$ is:

a) 1
b) 3
c) 5
d) 1/3
e) ∞

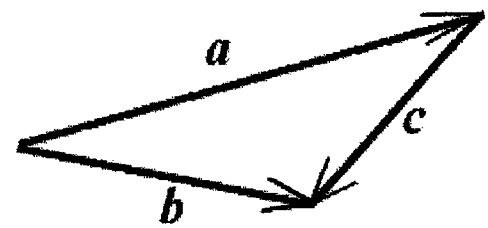
16. If the Taylor series for $f(x)$ centered at $a = 5$ is

$$-4 + 2(x-5) + \frac{7}{2}(x-5)^2 - \frac{1}{2}(x-5)^3 + \cdots$$

then the graph of f at $x = 5$ is:

a) increasing and concave up
b) increasing and concave down
c) decreasing and concave up
d) decreasing and concave down
e) none of these

17. For vectors $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ as drawn, which is correct?



a) $\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{c}$
b) $\boldsymbol{a}+\boldsymbol{c}=\boldsymbol{b}$
c) $\boldsymbol{b}+\boldsymbol{c}=\boldsymbol{a}$
d) $\boldsymbol{a}+\boldsymbol{b}+\boldsymbol{c}=0$
e) $\boldsymbol{a}\times\boldsymbol{b}=\boldsymbol{c}$

18. A vector perpendicular to both $\langle 3,1,2\rangle$ and $\langle 2,0,-3\rangle$ is:

a) $2\mathbf{i}-3\mathbf{k}$ b) $3\mathbf{i}+2\mathbf{k}$
c) $3\mathbf{i}-13\mathbf{j}+2\mathbf{k}$ d) $\langle 6,0,-6\rangle$
e) none of these

19. The line through the points
$(3,1,2)$ and $(3,2,-4)$
satisfies the parametric equations:

a) $x=3$, $y=1+t$, $z=2-6t$	b) $x=3+3t$, $y=1+2t$, $z=2-4t$	c) $x=3+3t$, $y=2+t$, $z=-4+2t$	d) $x=3+t$, $y=1+t$, $z=2+t$	e) $x=3t$, $y=1+t$, $z=-6+2t$

20. An equation of the plane containing the point $(3,-2,4)$ and orthogonal to the vector $4\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ is:

a) $3(x-4)-2(y-6)+4(z+2)=0$
b) $4(x-3)+6(y+2)-2(z-4)=0$
c) $4(x+3)+6(y-2)-2(z+4)=0$
d) $3(x+4)-2(y+6)+4(z-2)=0$
e) $-20(x-3)+22(y+2)+26(z-4)=0$

PART TWO. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES.
CALCULATORS PERMITTED FOR ALL PROBLEMS <u>EXCEPT</u> 27, 28, 29, 30.

21. a) Simpson's rule using n subdivisions is

$$S_n = \frac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+\ldots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)].$$

Use S_4 (Simpson's rule with 4 subdivisions) to approximate the area under one hump of the function $f(x)=\sin(x)$ over the interval $[\,0\,,\pi\,]$. Give your answer to 3 decimal places.

b) How much error is in your approximation?

22. A cylindrical swimming pool has a radius of 10 ft. The sides are 4 ft high and the depth of the water is 3 ft. How much work is required to pump all of the water out over the side? (Use the fact that the weight density of water is 62.5 lbs/ft^3.)

23. A simple closed circuit consists of an 8 Ohm resistor R, a 2 Henry inductor L, and a battery supplying a constant EMF $E(t) = 32$ Volts. If the initial current is 0 Amperes when the switch is closed, set up and use separation of variables to solve an initial value problem to determine the current $I(t)$ for $t > 0$. (HINT: Kirchhoff's law states that $L\frac{dI}{dt} + RI = E$.)

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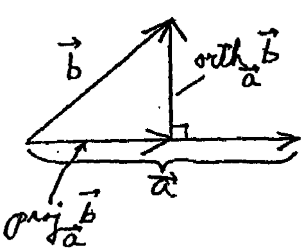
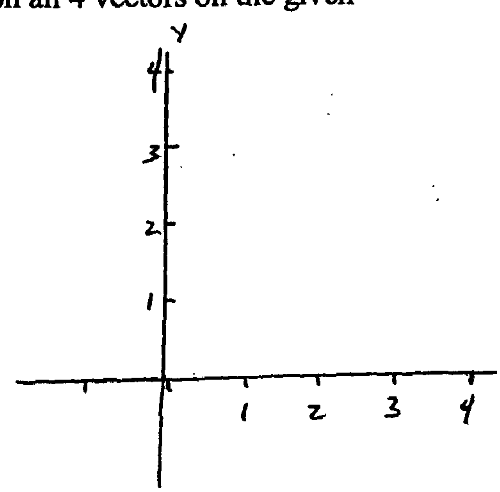
24. If $\vec{a}$ and $\vec{b}$ are vectors, (and $\vec{a} \neq \vec{0}$), then $\vec{b} - proj_{\vec{a}}\vec{b}$ is a new vector called $orth_{\vec{a}}\vec{b}$ (the projection of $\vec{b}$ orthogonal to $\vec{a}$).

a) Prove that $\vec{b} - proj_{\vec{a}}\vec{b}$ is perpendicular to $\vec{a}$. { HINT!: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ }



b) If $\vec{a} = \langle 3,3 \rangle$ and $\vec{b} = \langle 1,2 \rangle$, find $proj_{\vec{a}}\vec{b}$ and $\vec{b} - proj_{\vec{a}}\vec{b}$ and graph all 4 vectors on the given axes.



25. a) Use the Maclaurin series for $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$ to find the first four non-zero terms of the Maclaurin series for $\sin(x^2)$.

b) Use your first three non-zero terms from part a) to approximate $\int_0^1 \sin(x^2)\,dx$.

c) Give an upper bound for the error in your answer to part b).

26. The Mean Value Theorem for integrals says that if f is a continuous function on $[a, b]$, then there exists a number c in $[a, b]$ such that $f(c) = f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx$.

a) Find f_{ave} , the average value of $f(x) = \sqrt{x}$ on $[0,4]$.

b) Find a value of c such that $f_{ave} = f(c)$.

c) Sketch the graph of f on $[0,4]$ and shade a rectangle whose area is the same as the area under the graph of f.

CALCULATORS NOT PERMITTED FOR 27, 28, 29, 30.

27. a) Evaluate $\int_0^1 (x^4 - x^2 + x)\,dx$

b) Find the area of the region R bounded by the curves $y = x^2 - 2$ and $y = x$.

28. Evaluate $\int_0^1 \sqrt{(9x^3 + 16)}\,27x^2\,dx$ showing all steps.

29. Evaluate the following indefinite integrals showing all of your steps:

a) $\int x e^{3x} \, dx$

b) $\int \frac{3t+1}{(t+2)(t-3)} \, dt$

30. Use separation of variables to solve for y explicitly if $\frac{dy}{dx} = 3yx^2$, $y(0) = 5$.