Name____________________

Books and notes are closed. Calculators are open, loaded with any information you please; calculators may not be lent or borrowed. Intermediate work must be shown.

Hand this question sheet in with your answer booklets, and make sure each booklet has your name.

1 Solve the inequality $|x - 13| < 100$.

2a Define the convergence of the sequence $a_n$ to the limit $L$.
2b Prove that the sequence $a_n = \frac{1}{\sqrt{n}}$ converges and find the limit.
2c Use the squeeze theorem (sandwich theorem) to find the limit of $\left\{ \frac{\cos n}{\sqrt{n}} \right\}$, justifying your answer.

3 Set up the integral to find the area in the first quadrant between the curves $y = x^2$ and $y = \sqrt{x}$. Then solve.

4a Give the definition of “derivative” using limits. Use that definition to find the velocity if the position is $\sqrt{1 + t}$. No quick Newton formula, please.
4b Take the derivative with respect to $x$ of $\cos(x\cos x)$, using Newton’s formulas and methods.

5a If $y^3 + x^5 + xy = 3$, find an equation of the tangent to that curve at (1,1).
5b If displacement is given by $s(t) = 8 + \frac{1}{t} \sin(12\pi t)$, where $t$ is time, find velocity and acceleration.

6 Graph the curve $y = 3x^2 - 12x + 5$ on [0,3]. Show and label the intervals where it is increasing, decreasing, concave upward, or
concave downward; show and label any inflection points. Find the absolute maximum and absolute minimum on [0,3].

7 Show that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one root.

8 The radius of a right circular cylinder is increasing at six inches per minute. The height is two feet. How fast is the volume increasing when the radius is three feet?

9 Get an approximation to $\sqrt{10}$ using differentials (or, equivalently, using a linear approximation.)

10 Find the dimensions of a rectangle with area 500 square meters whose perimeter is as small as possible.

11a Evaluate $\int_2^3 x^{-5} + 17x + \sqrt{x} \, dx$.

11b Evaluate $\int_1^4 \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx$.

11c Evaluate $\int_0^{\pi/2} \frac{\sin x}{(1 + \cos x)^2} \, dx$.

12 Find the point on the line $y = x$ which is closest to the point (3,1).

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