

**For full credit, show your work. Points per problem: 1, 2, 4 ... 20 points; 3, 5, 6 ... 15 points.**

1. Consider the matrix  $A$  and the vector  $\vec{b}$  given by  $A = \begin{bmatrix} 1 & -1 & -5 \\ -3 & -7 & 5 \\ 0 & 1 & 1 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 7 \\ 29 \\ -5 \end{bmatrix}$ .

a. Solve the system  $A\vec{x} = \vec{b}$ .

b. If the system in (a) has more than one solution, list two of them.

2.  $A, B$  and the vector  $\vec{b}$  are given by  $A = \begin{bmatrix} 1 & 1 & -5 \\ -3 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 0 \\ -9 \\ 3 \end{bmatrix}$ .

a. Explain why the system  $A\vec{x} = \vec{b}$  has exactly one solution.

b. For the solution vector to the system  $A\vec{x} = \vec{b}$ , compute the value of its top row entry. Do NOT find all the entries of  $\vec{x}$ .

c. For each of the operations listed below, decide if it is well defined or undefined (you do NOT need to compute the operations). Cross out those undefined. Circle those defined, and next to them state if the result is scalar, or a matrix, and specify the dimensions of the matrix.

$\det A$

$AB$

$A + B$

$BA$

$BI_2$

$A^{-1} + A$

$B\vec{b}$

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3. For which value of  $m$  is the function  $y = t^m$  a solution of the differential equation  $t^2 y'' + \frac{1}{4}y = 0$ ?

- a)  $m = 1$       b)  $m = 1/2$       c)  $m = 1/4$       d)  $m = -1/4$       e)  $m = -1/2$ .

4. Solve the differential equation

$$\frac{dy}{dt} + \frac{2}{t}y = 4, \quad y(3) = 5.$$

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5. A tank contains 70 gallons of water with 10 lb of salt dissolved in it. Brine with concentration of 2 lb/gal is pumped into the tank at a rate of 2 gal/min. The well-stirred mixture is pumped out at the rate of 4 gal/min. A differential equation for the amount  $A$  (in lb) of salt in the tank is:

a)  $\frac{dA}{dt} = 10 - \frac{2}{35}A$    b)  $\frac{dA}{dt} = 4 - \frac{2}{35}A$    c)  $\frac{dA}{dt} = 10 - \frac{4}{70-2t}A$    d)  $\frac{dA}{dt} = 4 - \frac{2}{35-t}A$

6. CHOOSE ONLY ONE of the following problems (circle A, B or C):

Choice A: Find the general solution to  $e^t \frac{dy}{dt} = y^2 + 1$ .

Choice B: Use Euler's method with  $h = 0.5$  to approximate  $y(3)$ , if  $y$  satisfies;  $\frac{dy}{dx} = x + 2y + 2$ ,  $y(2) = -1$ .

Choice C: The differential equation  $\frac{dy}{dx} = 3y(5 - y)$  models some population growth.

a. Find all the equilibrium solutions.

b. Use the equilibrium (constant) solutions to identify which of the following graphs captures two solution curves:

