Exam 2, SM212P (Popovici), Monday Feb. 27, 2017 **SHOW WORK. No work, no credit.**

NAME:

Problem 1: Consider the differential equation

$$D^2(D^2 - 4D + 5)(D - 6)(D^2 + 9)^2[y(t)] = 4 + t^2e^{6t} + 8\sin(3t) + e^t\cos(3t)$$

(a) Find the form of the general homogeneous solution.

(b) Without determining any coefficients, determine the form for the particular solution.

Problem 2: Use undetermined coefficients to solve the initial value problem

$$y'' + 6y' + 8y = 12e^{-2x}, y(0) = -1, y'(0) = 2.$$

Problem 3: Given that $y = t^3$ and $y = t^5$ are linearly independent solutions of the homogeneous equation

$$t^2y'' - 7ty' + 15y = 0,$$

and that $y = \ln t + 0.125t$ is a particular solution of the non-homogeneous equation

$$t^2y'' - 7ty' + 15y = 15\ln t - 7$$

which of the following is the general solution of the non-homogenous equation?

a) $y = c_1 \ln t + c_2(0.125t) + c_3t^3 + c_4t^5$ b) $y = c_1(\ln t + 0.125t) + c_2t^3 + c_3t^5$

c) $y = \ln t + 0.125t + c_1t^3 + c_2t^5$ d) $y = c_1t^3 + c_2t^5$ e) $y = c_1 \ln t + c_2(0.125t)$.

Problem 4. A spring without damping is set in motion and oscillates with displacement

$$y(t) = 3\cos(4t) - 3\sin(4t).$$

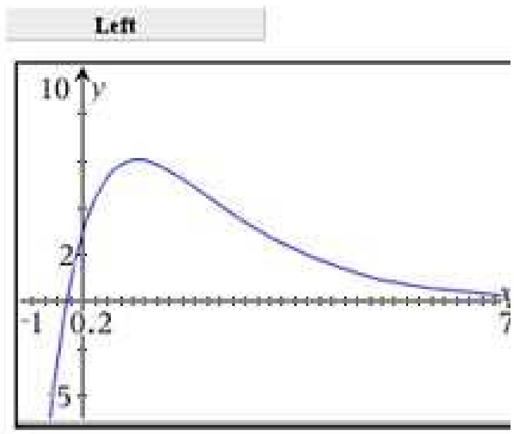
a. What is the amplitude of the motion?

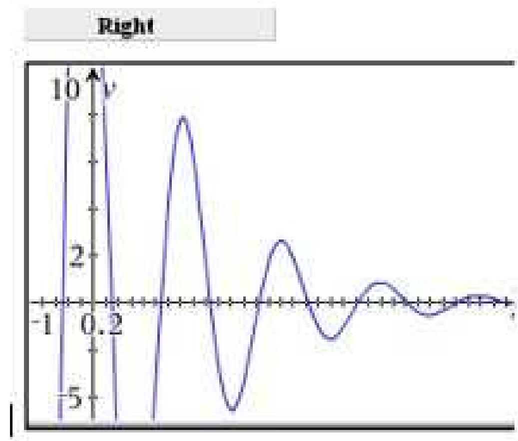
b. What is the period of the motion?

c. What angular frequency should a driving force $F_0 \cos(\gamma t)$ have, if applied to the the mass-spring system, it is to induce resonance?

d. Suppose that the mass was $m = 1$Kg. What is the spring constant k? Give example of a damping constant that will make the system under-damped.

Problem 5. The deformation y of a damped spring is described by $y'' + 6y' + ky = 0$, where k is the spring constant. The spring motion is OVER-damped. Select the graph of y and the value of k from the options below that are consistent with OVER-damping.





a) Left graph, $k = 25$ b) Left graph, $k = 8$ c) None of the graphs works

d) Right graph, $k = 25$ e) Right graph, $k = 8$.

Problem 6: An RLC system with $L = 1\text{h},\ R = 2\ \Omega,\ C = 1/2\, C$ is connected to a source $E(t) = \sin t\ V$. The charge settles into a steady state solution.

a. Set up a differential equation satisfied by the charge on the capacitor.

b. Find the steady state for the charge.