NAME:
SM212P
April 26, 2017

SHOW WORK.
NO CALCULATORS.

Problems 1,3,4,5 are 20 points each. Problem 2 is 15 points. The extra 5 points are for being neat, submitting work that is organized and easy to read.

1. Rewrite the system of differential equations given below in matrix notation. Use eigenvalues and eigenvectors to find the general solution.

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = 5x + 4y$$

2. Rewrite the differential equation $y'' + 4y' - 8y = 4$ with initial condition $y(0) = 1,\ y'(0) = 0$ as a system consisting of two first-order equations. Use Euler's method with $h = 0.25$ to approximate $y(0.5)$. Make sure to clearly label your variables/table, etc.

3. Consider the periodic function f, given on its main period as $f(x) = \begin{cases} 0 & \text{for } -2\pi < x \leq 0, \\ x & \text{for } 0 < x \leq 2\pi. \end{cases}$

a. Sketch the graph of the Fourier Series (denoted FS) over an interval that extends to include at least two periods. Label your axes, and the important points of the graph.

b. To what values does the FS converge when $x = -2\pi,\ \ x = 0,\ \ x = \pi$, respectively. Answer by filling in :

$FS(-2\pi) =$ $\qquad$ $FS(0) =$ $\qquad$ $FS(\pi) =$ $\qquad$ BONUS: $FS(-8) =$

c. Some of the Fourier series coefficients have been computed. They are: $a_1 = -4/\pi,\ \ a_2 = 0,\ \ b_1 = 2$. Compute a_0 and b_2; simplify your answers. You may want to use the antiderivatives provided in the tables.

$a_0 =$

$b_2 =$

d. Use all the information from 3c the write the first few terms of the Fourier series. You do NOT have to find the general term.

4. Consider the following heat equation problem, with zero ends:

$$\frac{\partial u}{\partial t} = 0.08\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < \pi, \ \ t > 0 \qquad \text{and } \ u(0,t) = u(\pi,0) = 0.$$

a. Find the general solution, $u(x,t)$. You do NOT have to show the separation of variables.

b. Solve the problem if initially the temperature distribution was $u(x,0) = f(x) = \begin{cases} 100 & \text{for } 0 < x \leq \pi/2, \\ 0 & \text{for } \pi/2 < x \leq \pi. \end{cases}$

5. Solve ONE of the following two problems. The one you skip is HOMEWORK due Friday, April 28, start of class.

OPTION A (faster) : Find all solutions $U(x, y)$ of product type for the differential equation

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 5U.$$

Show all the steps in the separation of variables.

OPTION B: Find all solutions $u(x, t)$ of product type for the heat equation with zero ends

$$\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < 2, \;\; t > 0 \qquad \text{and} \;\; u(0, t) = u(2, 0) = 0.$$

Show the separation of variables, and show how to solve the separated equations in order to get u *only* in the case compatible with the zero ends condition. It is NOT enough to give the solution u using the provided formula.

NAME: OPTION DUE ____

The two problems given below were part of Wednesday's exam. You got to solve one during the exam. The remaining one is assigned as homework; it is due April 28, start of class.

OPTION A: Find all solutions $U(x, y)$ of product type for the differential equation

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 5U.$$

Show all the steps in the separation of variables.

OPTION B: Find all solutions $u(x, t)$ of product type for the heat equation with zero ends

$$\frac{\partial u}{\partial t} = 8\frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < 2, \ \ t > 0 \qquad \text{and} \ \ u(0, t) = u(2, 0) = 0.$$

Show the separation of variables, and show how to solve the separated equations in order to get u *only* in the case compatible with the zero ends condition. It is NOT enough to give the solution u using the provided formula.