

SM219 Final Exam

This exam is “open book and open notes”. You may use your graphing calculator; you are not to borrow a sectionmate’s calculator. You are to reduce numerical answers to simple fractions or decimal fractions. Please show all work and answers on the blank solution sheets provided. “Box” your answers. You are to hand in these question sheets along with your solution sheets; be sure your name is on each sheet.

Name: _____ Section: _____

For all tests of hypotheses, report the P-value and base your conclusion on a level of significance $\alpha = 0.05$.

1. There is a bag of 5 fresh AA batteries for “calculator emergencies” stored on the company wardroom table. Unfortunately, some plebe comes along and drops in 4 dead AA batteries, thinking the bag is for recycling. On your way to your SM219 exam, you notice that your TI92 has gone dead. You know you’re behind schedule, so you plan on swapping out your 4 old batteries when you get to there. You grab 4 batteries at random from the bag of 9 as you rush to the exam. The number of dead batteries X among these 4 has the following probability distribution:

x	0	1	2	3	4
p(x)	0.040	0.317	0.476	0.159	0.008

- (a) What is the probability that you have at least one dead battery?
- (b) Sketch the probability histogram for this distribution.
- (c) Calculate the mean μ_X of X .
- (d) Calculate the mean of the random variable $4-X$.
- (e) You are given that the standard deviation of X is $\sigma_X = 0.7857$. Calculate the standard deviation of the random variable $4-X$.

2. Assume that the average lifetime X in hours of a Duracell Alkaline AA battery has mean $\mu_X = 4.1$ and standard deviation $\sigma_X = 1.8$. Assume that the average lifetime Y in hours of an Eveready Energizer Alkaline AA battery has mean $\mu_Y = 4.4$ and standard deviation $\sigma_Y = 1.9$. Furthermore, assume X and Y have normal distributions.

- (a) Calculate the probability that one Duracell battery will last longer than 6.0 hours.
- (b) Calculate the probability that one Eveready battery will last longer than 6.0 hours.
- (c) Assume that an SRS of 10 Eveready batteries is selected. Calculate the probability that 3 or fewer will last longer than 6.0 hours.

Let $T = X + Y$ be the total lifetime of one Duracell and one Eveready battery. Assume the two batteries operate independently of one another.

- (d) Calculate the mean of T .
- (e) Calculate the standard deviation of T .
- (f) Calculate the probability that the total lifetime of the two batteries is more than 12 hours.

3. From a recent AP story (reported in *The Capital*, 20 April 2003) titled “Salmonella Prevalence Decreases”:

“Of the 58,085 samples of meat and poultry tested for salmonella last year, 4.3% [2498 samples] had the germ which can cause food poisoning, the report says. That’s down from two years ago when 5% [2297 samples] of the 45,941 samples of meat and poultry tested positive.”

- (a) Construct a 99% confidence interval for the proportion of last year’s samples that had germs.
- (b) Test whether this decrease in proportion is statistically significant.

4. Fifteen air samples from a certain region were obtained and for each one the carbon monoxide concentration was determined. The results (in ppm) were:

CO	9.3	10.7	8.5	9.6	12.2	15.6	9.2	10.5	9.0	13.2	11.0	8.8	13.7	12.1	9.8
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- (a) Draw a stemplot with stems: 8, 9, 10, 11, 12, 13, 14, 15.
- (b) Give the Five-Number Summary for this set of observations.
- (c) Calculate the sample mean \bar{x} for this data.
- (d) Calculate the sample standard deviation s for this data.
- (e) Calculate the Interquartile Range IQR .
- (f) Use IQR to identify any suspected outliers.
- (g) Sketch a modified boxplot for this set of observations.
- (h) What percent of these 15 observations lie between the numbers $\bar{x} - 2s$ and $\bar{x} + 2s$?

5. A manufacturer is developing a nickel-metal hydride battery that is to be used in cellular telephones in lieu of nickel-cadmium batteries. The director of quality control decides to evaluate the newly developed battery against the widely used nickel-cadmium battery with respect to performance. A random sample of 25 nickel-cadmium batteries and a random sample of 25 of the newly developed nickel-metal hydride batteries are placed in cellular telephones of the same brand and model. The performance measure of interest is the talking time in minutes prior to recharging. The results of the experiment follow:

	n	\bar{x}	s
Metal	25	79.73	15.03
Cadmium	25	70.75	13.99

- (a) Construct a 95% confidence interval for the mean talking time prior to recharging μ_M of the nickel-metal hydride battery.
- (b) Is there evidence of a difference between the two types of batteries with respect to mean talking time prior to recharging? Specifically, does μ_M equal μ_C ? Give a complete significance test: state null and alternate hypotheses, calculate the appropriate statistic, determine the P -value for the test, and state your conclusion
- (c) Construct a 95% two-sided confidence interval for the difference $\mu_M - \mu_C$ in mean talking time prior to recharging for the two types of battery.

6. In some locations there is a strong association between concentrations of two different pollutants. The paper “The Carbon Component of the Los Angeles Aerosol: Source Apportionment and Contributions to the Visibility Budget” (*J. Air Pollution Control Fed.*, 1984, pp. 643–650) reported the accompanying data on ozone concentration x (ppm) and secondary carbon concentration y ($\mu\text{g}/\text{m}^3$):

x	0.066	0.088	0.120	0.050	0.162	0.186	0.057	0.100
y	4.6	11.6	9.5	6.3	13.8	15.4	2.5	11.8

x	0.112	0.055	0.154	0.074	0.111	0.140	0.071	0.110
y	8.0	7.0	20.6	16.6	9.2	17.9	2.8	13.0

- (a) Calculate the sample correlation r for this data.
- (b) Assume the ozone concentration random variable X and the secondary carbon concentration random variable Y are normal with population correlation ρ . Test the hypothesis $H_0: \rho = 0$ against $H_a: \rho \neq 0$.
- (c) Calculate the equation of the simple regression line $\hat{y} = b_0 + b_1 x$. Carefully sketch this line in the scatterplot below.
- (d) Use your equation for the sample regression line to approximate secondary carbon concentration y for an ozone concentration of $x=0.130$. Carefully mark this point (x,y) in the scatterplot below.

