1330-1630 Monday 15 December 2008
Name ________________ Alpha Code ______________ Section ____________

This exam is composed of 3 parts. Part A. Manual Computation (20%). There are four problems. For this part of the exam, you will not be allowed to use your calculator. Do this part first. When you are finished, turn it in to your instructor. You may then take out your calculator and use it for the remainder of the exam. Part B. Multiple Choice (30%). Part C. Problem Solving (50%). You are allowed to use a formula sheet. It may be up to 8.5 by 11 inches. You may use both sides. However, the information must be filled in by you by hand. No duplication of another person's formula sheet either using a copier or electronically.

Part A. Manual Computation. For these 4 problems, **you are not allowed to use a calculator**. When you are finished, turn in this sheet to your instructor. You may then use your calculator for the remainder of the exam.

A1. Given $\vec{\mathbf{r}}(t) = t\hat{\mathbf{i}} + t\cos(t)\hat{\mathbf{j}} + t\sin(t)\hat{\mathbf{k}}$.

a) Find the velocity at t=0.

b) Find the speed at t=0.

c) Find the acceleration at t=0.

d) Sketch the curve. Be sure to label your axes.

A2. For the function, $f(x,y) = x^2 + y^3 - xy$,

a) Find $f_x(x,y)$. b) Find $f_y(x,y)$.

c) Find the gradient at the point $(1,1), \nabla f(1,1)$.

d) Find the directional derivative at the point $(1,1)$ in the direction of the vector <4,-3>.

e) Find a unit vector in the direction of greatest increase of the function at the point $(1,1)$.

A3. Compute $\iint_D x^2 dA$ where D is bounded by $x = 0$, $y = 0$, $y = 1 - x$.

A4. Use Green's Theorem to compute $\int_C -x^2 y dx + xy^2 dy$ where C is the unit circle, $x^2 + y^2 = 1$ traversed counterclockwise.

End of Part A. Turn in this sheet to your instructor. Use may use your calculator for the reminder of the exam

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Part B. Multiple Choice. You may use a calculator on this portion of the exam. Choose the best answer to the problem. Circle the correct answer on the exam page and bubble in the correct your answer on the bubble sheet. Show any supporting work that you did to come up with the answer on the exam page or on a separate sheet of paper available.

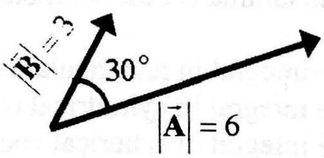
B1. The distance from the point (-12,4,3) to the *yz* plane is

a) 12 b) -12 c) 5 d) -5 e) 25

B2. The vectors are in the *xy* plane. $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is

a) 9 b) $9\sqrt{3}$ c) $9\hat{\mathbf{k}}$

d) $-9\hat{\mathbf{k}}$ e) $9\sqrt{3}\hat{\mathbf{k}}$



B3. If $\vec{\mathbf{v}}(t) = \langle \cos(t), \sin(t), t \rangle$ and $\vec{\mathbf{r}}(0) = \langle 3,2,-1 \rangle$ then $\vec{\mathbf{r}}(t) =$

a) $\langle \sin(t)+3, \cos(t)+2, t^2-1 \rangle$ b) $\langle -\sin(t)+3, \cos(t)+2, t^2/2+1 \rangle$

c) $\langle -\sin(t), \cos(t), 1 \rangle$ d) $\langle \sin(t)+3, -\cos(t)+2, t^2/2-1 \rangle$

e) $\langle \sin(t)+3, -\cos(t)+3, t^2/2-1 \rangle$

B4. Let $g(x,y) = x^2y + x^2 + y$. Then $g_x(1,2) =$

a) 4 b) 6 c) 7 d) $2xy+2x$ e) x^2+1

B5. The level surfaces of $f(x,y,z) = x^2 + y^2$ are a family of

a) spheres b) cones c) cylinders d) paraboloids e) hyperboloids

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B6. If $\nabla f(x_0, y_0) = 5\hat{\mathbf{i}} - \hat{\mathbf{j}}$ then the directional derivative at (x_0, y_0) in the direction of $8\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$ is

a) 34, b) 4.6 c) 3.4 d) $40\hat{\mathbf{i}} - 6\hat{\mathbf{j}}$ e) $4\hat{\mathbf{i}} - 0.6\hat{\mathbf{j}}$

B7. For the surface $z = x^2 + y^2$, the equation of the plane tangent to the surface at the point (2,1,5) is

a) $z+5 = 4(x+2)+2(y+1)$ b) $z+5 = 2x(x+2)+2y(y+1)$ c) $z = 2x+2y$

d) $z-5 = 2x(x-2)+2y(y-1)$ e) $z-5 = 4(x-2)+2(y-1)$

B8. $\iint_D \left(x^2+y\right)dA$, where D is the area bounded by $x=0,\ x=2,\ y=0,\ y=x^2$, is closest to

a) 1 b) 3 c) 8 d) 10 e) 27

B9. A lamina consists of the region in the first quadrant inside the circle $x^2+y^2=4$ and outside the circle $x^2+y^2=1$. If the density is given by $\rho(x,y)=\dfrac{1}{x^2+y^2}$, the mass of the lamina is best computed using

a) A triple integral in rectangular coordinates
b) A triple integral in cylindrical coordinates
c) A triple integral in spherical coordinates
d) A double integral in rectangular coordinates
e) A double integral in polar coordinates.

B10. The volume of a solid bounded by $z=\sqrt{x^2+y^2}$ and $z=4$ is closest to

a) 12π b) 16π c) 64π d) $64\pi/3$ e) $128\pi/3$

B11. $\iiint_E \left(x^2+y^2+z^2\right)dV$ where E is the solid $x^2+y^2+z^2\le 4,\ z\ge 0$, is closest to

a) 0 b) $\dfrac{8\pi^2}{3}$ c) $\dfrac{64\pi}{5}$ d) $\dfrac{128\pi}{5}$ e) $\dfrac{2048\pi}{5}$

B12. $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$, where $\vec{\mathbf{F}}(x,y,z)=\langle x,z,y\rangle$ and C is the line segment from (0,0,0) to (2,3,4), is closest to

a) 14 b) 14.5 c) 0 d) 5 e) 10

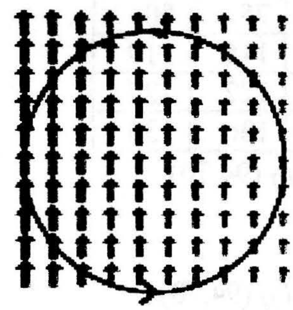
B13. $curl(div(f(x,y,z)))$

a) takes a scalar function to a scalar function
b) takes a scalar function to a vector function
c) takes a vector function to a scalar function
d) takes a vector function to a vector function
e) is not defined

B14. $\iint_S \vec{\mathbf{F}}\cdot d\vec{\mathbf{S}}$ where S is the surface $x^2+y^2=4,\ 0\le z\le 3$, $\vec{\mathbf{F}}(x,y,z)=\langle x,y,z\rangle$ oriented outward is closest to

a) -24π b) 0 c) 24π d) 36π e) -36π

B15. The work done by the vector field as a particle moves counterclockwise around the closed path is



a) positive
b) 0
c) negative
d) a vector point out of the paper
e) a vector point into the paper

End of Part B.

Part C. Problem Solving. Work these 10 problems on the exam paper. You may use extra paper, if you need it. You may use your calculator.

C1. Given the points $P(3,1,4)$, $Q(1,-2,1)$, $R(1,4,-2)$ and $S(1,2,1)$.
a) Find the equation of the plane containing the points P, Q and R.

b) Find equations of the line through the point S which is perpendicular to the plane in part a).

C2. At the end of an important football game, a field goal kicker attempts a 50 yard (150ft) field goal. He kicks the ball at an angle of elevation of 45 degrees and an initial speed of 72 ft/sec. What is the height of the ball when it crosses the goal posts?

C3. The table below gives the heat index, $I\,(T,H)$ as a function of temperature and humidity.

$T\backslash H$	70	75	80
92	112	115	119
94	118	122	127
96	125	130	135

a) Estimate $I_T(94,70)$.

b) Estimate $I_H(94,70)$.

c) Use a linear approximation to estimate $I(95, 72)$.

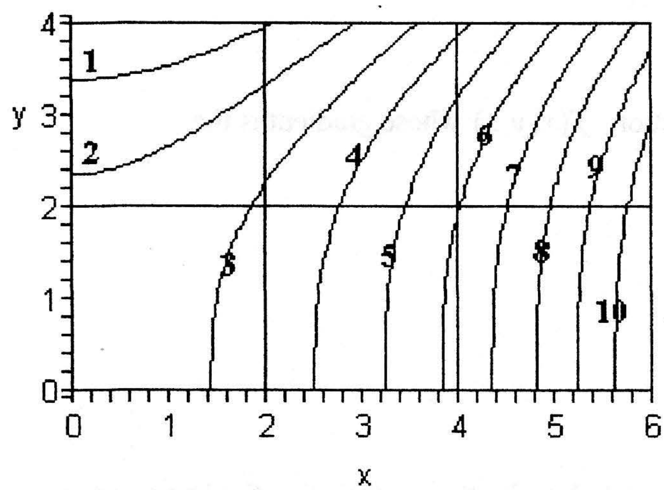
d) If the temperature is 94 degrees and increasing at the rate of 3 degrees per hour and the humidity is 70 percent and decreasing at the rate of 2 percent per hour, use the chain rule to estimate the rate of change of the heat index with respect to time.

C4. For the function, $f(x,y) = 3x - x^3 + y^3 - 12y$.

a) Find the critical points

b) For each critical point, determine whether it is a local maximum, local minimum or a saddle point. You may use the formula $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.

C5. A contour map is shown for the function, g, on the square $R=[0,6]\times[0,4]$. Use a Riemann sum with m=3 and n=2 with **evaluation at the upper right corner** of each rectangle to estimate $\iint_R g(x,y)dA$



C6. Compute $\iiint_E \left(x^2+z\right)dV$ where E is the solid tetrahedron bounded by the coordinate planes and the plane $2x+2y+z=2$

C7. Find $\iiint_E \left(x^2+y^2+z^2\right)dV$ where E is the solid cylinder $x^2+y^2\le 9, 0\le z\le 2$

C8. $\vec{\mathbf{F}}_1(x,y,z) = \langle 2xy+1,\ 2x,\ x+2z \rangle$, $\vec{\mathbf{F}}_2(x,y,z) = \langle x^2+y,\ x+z,\ y+z^2 \rangle$.

a) One of these vector fields is conservative. Determine which one.

b) For the one that is, find a scalar function, $f(x,y,z)$ whose gradient is the conservative vector field.

c) Use the answer to part b) to find the work done by the conservative field as a particle moves from (0,0,0) to (1,-1,1).

C9. Use the Divergence Theorem to compute $\iint_S \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where S is the surface that bounds the region $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 3$ and $\vec{\mathbf{F}}(x,y,z) = \langle 2x, 2y, z^2 \rangle$, oriented outward.

C10. Use Stokes' Theorem to compute $\iint_S \nabla \times \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$ where S is given by $z = 4 - x^2 - y^2$, $z \geq 0$ and $\vec{\mathbf{F}}(x, y, z) = \langle y, -x, z^2 \rangle$ oriented upward.

End of Part C.

End of the Exam

Male sure your alpha code is bubbled in on the bubble sheet. Make sure your name and alpha code are on the front page of this exam, any seperate sheets of paper you used and on your formula sheet. Turn all of these in to your instructor