SM221-3041

NAME: ____________________________ **ALPHA NU**DARCHANGELO JAMES M

INSTRUCTOR: ____________________________ **SECTION:** 20

FALL 2009

CALCULUS III (SM221, SM221P) **FINAL EXAMI**

1330-1630 Wednesday 16 Dec 2009 SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor and section on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS PERMITTED FOR THIS SECTION.

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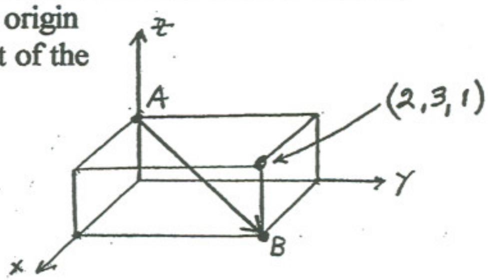
1. The rectangular box shown on the right has one vertex at the origin and the opposite vertex at $(2,3,1)$. (The positive x-axis comes out of the page toward you.) The vector $\overrightarrow{AB}$ in the sketch is:



a) $<2,3,1>$ b) $-2\hat{i}+3\hat{j}+\hat{k}$ c) $<0,2,1>$

d) $2\hat{i}+3\hat{j}-\hat{k}$ e) $<2,-3,0>$

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2. Find x so that the vectors $<6,x,3>$ and $<6,-7,x>$ are perpendicular.

a) 0 b) 2 c) 5

d) -9 e) 9

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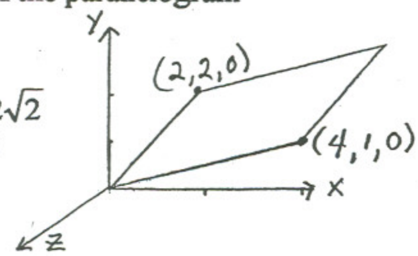
3. The area of the parallelogram on the right is:



a) $\sqrt{2}$ b) $2\sqrt{2}$

c) 4 d) 6

e) $6\sqrt{2}$

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4. If f is a scalar field and $\vec{F}$ is a vector field, which of the following expressions results in a scalar field?

a) $div\vec{F}$ b) $grad(div\vec{F})$ c) $div\,f$ d) $curl\,\vec{F}$ e) none of these

5. The contour curves (level curves) in the xy-plane for the function $f(x,y)=\sqrt{x^2+y^2}$ are:

a) paraboloids b) cones c) circles d) right triangles e) spheres

6. An equation for the tangent plane to the surface $z=e^{3x}+\sin(y)$ at the point (0,0,1) is:

a) $z=3x+2$ b) $3x+y+z=1$
c) $z=1+3x+y$ d) $x-y+z=0$
e) $z=y$

7. The maximum directional derivative for the function $f(x,y)=xy^2$ at the point (2 , 3) is:

a) 288/5 b) 1
c) 6 d) $12\sqrt{3}$
e) 15

8. When we reverse the order of integration for the iterated integral $\int_0^1\int_0^{x^2} e^y\,dy\,dx$, we get:

a) $\int_0^1\int_{\sqrt{y}}^1 e^y\,dx\,dy$

b) $\int_0^1\int_{\sqrt{y}}^1 \ln(x)\,dx\,dy$

c) $\int_0^{x^2}\int_0^1 e^y\,dx\,dy$

d) $\int_0^1\int_0^{y^2} \ln(x)\,dx\,dy$

e) $\int_0^1\int_0^{\sqrt{y}} e^y\,dx\,dy$

9. When we convert the iterated integral $\int_0^3 \int_0^{\sqrt{9-x^2}} \sin(x^2+y^2)\,dy\,dx$

from rectangular to polar coordinates, we get:

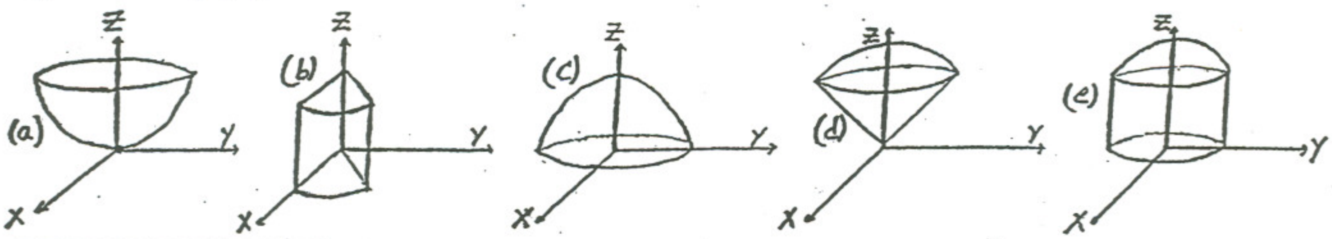
a) $\int_0^{\pi/2} \int_0^3 \sin(r^2)\,dr\,d\theta$ b) $\int_0^{2\pi} \int_0^3 \sin(r^2)\,r\,dr\,d\theta$

c) $\int_0^{\pi/2} \int_0^3 \sin(r^2)\,r\,dr\,d\theta$ d) $\int_0^{\pi} \int_0^{\sqrt{9-(r\cos(\theta))^2}} \sin(r^2)\,r\,dr\,d\theta$

e) $\int_0^3 \int_0^{\sqrt{9-(r\cos(\theta))^2}} \sin^2(r)\,dr\,d\theta$

10. What shape is the solid of integration E in the triple integral

$$\iiint_E 1\,dV = \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^3 \rho^2 \sin(\phi)\,d\rho\,d\phi\,d\theta\ ?$$

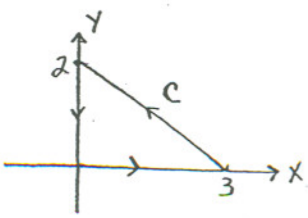


11. Suppose that $\vec{r}\,'(t) = <2e^{2t}, 3t^2, \cos(t)>$, and $\vec{r}(0) = <1,1,0>$. Find $\vec{r}(1)$.

a) $<e^2, 2, \sin(1)>$
b) $<2e^2+2, 2, 0>$
c) $<e^2+1, 2, \sin(1)>$
d) $<8e^2, 3, \sin(1)>$
e) $<8e^2, 4, -\sin(1)>$

12. Use Green's Theorem to evaluate the line integral
$\int_C 4y\,dx + (5x+y)\,dy$ where C is the closed curve boundary of
the triangle below, oriented counterclockwise.

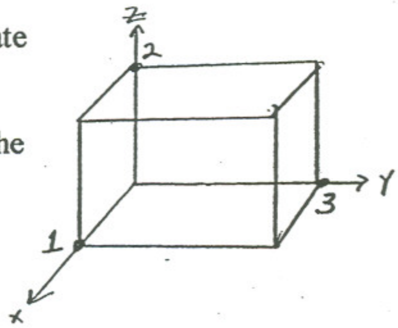
a) 0 b) 1
c) 2 d) 3
e) 6



13. The length of the curve $\vec{r}(t) = < t, t^2, t^3 >$ between the points where $t = -1$ and $t = 1$ is closest to :

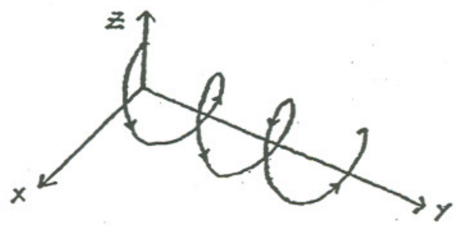
a) 1.3
b) 2.7
c) 3.4
d) 3.7
e) 4.0

14. Use the Divergence Thm $\left[\iint_S \vec{F} \cdot d\vec{S} = \iiint_E div\,\vec{F}\, dV\right]$ to calculate the flux $\left[\iint_S \vec{F} \cdot d\vec{S}\right]$ if $\vec{F}(x,y,z) = < 5x + y, z - y, x + y >$ and S is the boundary surface (6 faces oriented outward) for the box E shown on the right.



a) 0
b) π
c) π^2
d) 6
e) 24

15. The path on the right is the graph of what function?



a) $z = \cos(x) + \sin(y)$ for $0 \le x \le 6\pi$ and $0 \le y \le 6\pi$
b) $\vec{r}(t) = < \cos(t), \sin(t), t >$ for $0 \le t \le 6\pi$
c) $\vec{r}(u,v) = < \cos(u), \sin(u), v >$ for $0 \le u \le 6\pi$ and $0 \le v \le 1$
d) $\vec{r}(t) = < \sin(t), t, \cos(t) >$ for $0 \le t \le 6\pi$
e) none of a - d

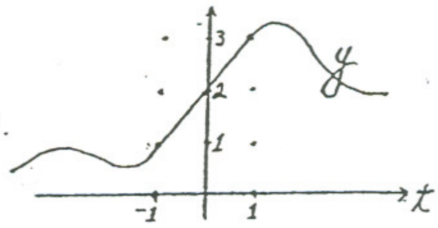
16. The function $f(s,t)$ is given by the table on the right. $\frac{\partial f}{\partial s}(1,2)$ is approximately:

s \ t	0	1	2	3
0	8	5	2	-1
1	9	7	5	3
2	10	9	8	7
3	12	11	10	10

a) -3
b) 3
c) -2
d) 2
e) 1

17. Let $z = x^3 + y^2$, where $x = e^t$ and y is a function of t graphed on the right.

Use the chain rule to approximate $\dfrac{dz}{dt}$ at $t = 0$.



a) 1 b) 4
c) 0 d) 7
e) 12

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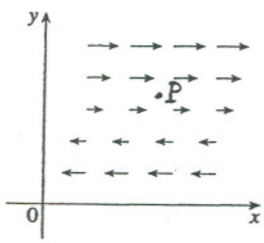
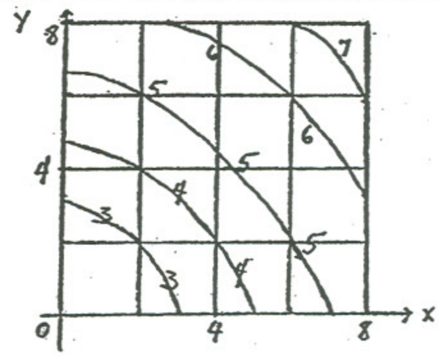
18. The vector field $\vec{F}$ is shown in the xy-plane and looks the same in all other horizontal planes. What statements are true for $\vec{F}$ at the point labeled P?

a) $div\,\vec{F} < 0,\ curl\,\vec{F} \neq \vec{0}$
b) $div\,\vec{F} = 0,\ curl\,\vec{F} = \vec{0}$
c) $div\,\vec{F} > 0,\ curl\,\vec{F} = \vec{0}$
d) $div\,\vec{F} = 0,\ curl\,\vec{F} \neq \vec{0}$
e) $div\,\vec{F} > 0,\ curl\,\vec{F} \neq \vec{0}$



Use the contour map on the right for the function $f(x, y)$ to solve problems 19 and 20.



19. Approximate $\int_0^8\int_0^8 f(x,y)\,dx\,dy$ by subdividing the region into four equal squares (i.e., n=m=2) and using midpoints to compute a Riemann Sum.

a) 16 b) 76 c) 100 d) 224 e) 304

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20. Evaluate the line integral $\int_C (\vec{\nabla} f)\cdot d\vec{r}$, where C is the curve given parametrically by $x = t^2 + t$; $y = 4t - 2$; where $1 \le t \le 2$, and the level curves of f are shown above.

a) 0 b) 1 c) 3 d) 4 e) 5

PART TWO. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES. **CALCULATORS PERMITTED FOR ALL PROBLEMS <u>EXCEPT</u> 27, 28, 29, 30.**

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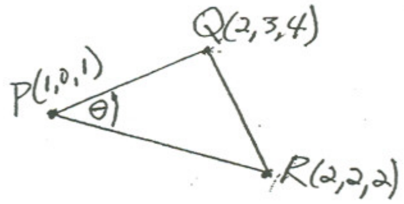
21. Let $P(1,0,1)$, $Q(2,3,4)$, and $R(2,2,2)$ be three points in space.



a) Find the angle θ between the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$.

b) Find an equation for the plane going through the three points.

c) Find parametric equations for the line going through P and Q.

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22. For the function $f(x,y) = x^3y + 12x^2 - 8y$,

a) find the critical points.

b) For each critical point, determine whether it is a local maximum, a local minimum, or a saddle point. Use the second derivative test with $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$.

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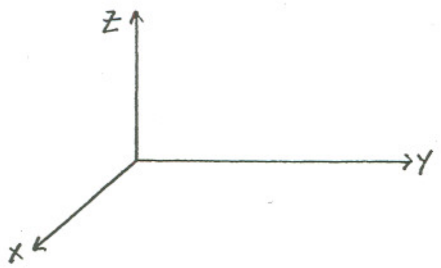
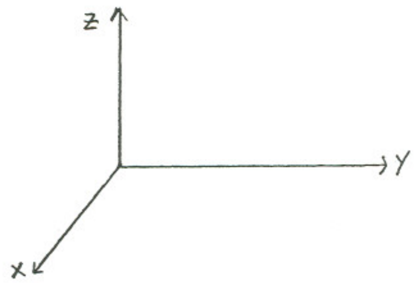
23. Sketch separate graphs in R^3, 3-d space, represented by each of the following equations:

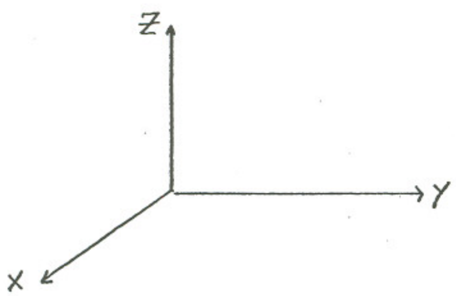
a) $x + 2y + 3z = 6$



b) $y = x$

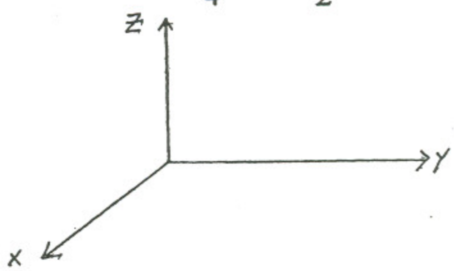


c) $z = x^2 + y^2$



d) Sketch the solid in 3-d space whose cylindrical coordinates satisfy the inequalities:

$2 \le r \le 4, \quad \frac{\pi}{4} \le \theta \le \frac{\pi}{2}, \quad 0 \le z \le 2.$



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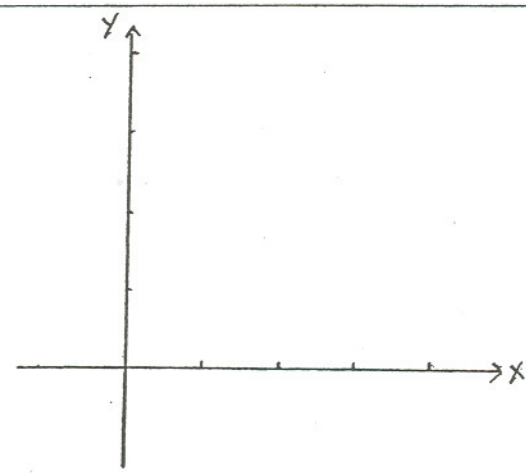
24. A soccer player kicks a ball from the ground toward the center of an empty goal (no goalkeeper). The ball leaves the ground going 50 ft/sec making an angle of $60°$ with the ground. If he takes the shot from 60 feet in front of the goal and the top of the goal is 8 ft high, will the ball go into the goal in the air? (Ignore air resistance.) Show the acceleration of the ball $\vec{a}(t)$, its velocity $\vec{v}(t)$, its position $\vec{r}(t)$, and solve the problem.

25. Consider the function $f(x,y) = y - \sqrt{x}$.

a) Sketch the level (contour) curves where $f(x,y) = 0$; $f(x,y) = 1$; and $f(x,y) = 2$.



b) Find $\vec{\nabla} f(1,1)$, (gradient of f at $(1,1)$), and draw it on your graph from part a).

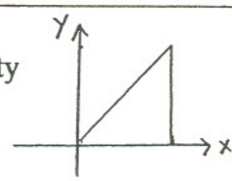
c) Find the directional derivative of f at $(1,1)$ in the direction $4\hat{i} + 3\hat{j}$.

26. a) If $f(x,y,z)$ is any scalar field (with continuous second partial derivatives), prove that $curl(grad\ f) = <0,0,0>$ (i.e., $\vec{\nabla} \times \vec{\nabla} f = <0,0,0>$).

b) Find a potential function $f(x,y,z)$ for the vector field $\vec{F}(x,y,z) = <y, x+z, y+2z>$.

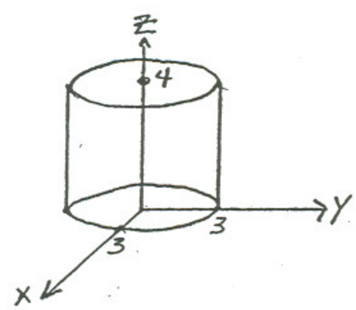
CALCULATORS NOT PERMITTED FOR 27, 28, 29, 30.

______ 27. Use double integrals to find $\bar{x}$, the x-coordinate for the center of mass, for the thin triangular plate with vertices $(0,0)$, $(1,0)$, and $(1,1)$, if the density of the plate (mass/area) is given by $\rho(x,y) = y$.



28. a) Evaluate the triple integral $\int_0^3 \int_0^2 \int_0^1 8xyz \, dz \, dy \, dx$.

b) Evaluate a triple integral to find the mass of the right circular cylinder on the right if it has density $\rho(x,y,z) = x^2 + y^2$ (mass/vol).



29. a) Evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$, where $\vec{F}(x,y,z) = \langle z,x,y \rangle$ and C is the line from (1,1,1) to (3,4,5).

b) Is the value for the line integral in part a) the same for any path from (1,1,1) to (3,4,5)? Explain.

30. Evaluate a surface integral to find the flux, $\iint_S \vec{F}\cdot d\vec{S}$,

where $\vec{F}(x,y,z) = \langle x,y,z \rangle$ and where S is the portion of the surface $z = 9 - x^2$ over the rectangular region $R = [0,2]\times[0,3]$ oriented upward. (See the figure on the right.)