1. The rectangular box shown on the right has one vertex at the origin and the opposite vertex at (2,3,1). (The positive x-axis comes out of the page toward you.) The vector $\overrightarrow{AB}$ in the sketch is:
   a) $<2,3,1>$  
   b) $-2\hat{i} + 3\hat{j} + \hat{k}$  
   c) $<0,2,1>$  
   d) $2\hat{i} + 3\hat{j} - \hat{k}$  
   e) $<2,-3,0>$

2. Find $x$ so that the vectors $<6,x,3>$ and $<6,-7,x>$ are perpendicular.
   a) 0  
   b) 2  
   c) 5  
   d) -9  
   e) 9

3. The area of the parallelogram on the right is:
   a) $\sqrt{2}$  
   b) $2\sqrt{2}$  
   c) 4  
   d) 6  
   e) $6\sqrt{2}$

4. If $f$ is a scalar field and $\vec{F}$ is a vector field, which of the following expressions results in a scalar field?
   a) $\text{div} \vec{F}$  
   b) $\text{grad} \left( \text{div} \vec{F} \right)$  
   c) $\text{div} f$  
   d) $\text{curl} \vec{F}$  
   e) none of these
5. The contour curves (level curves) in the xy-plane for the function \( f(x, y) = \sqrt{x^2 + y^2} \) are:

a) paraboloids  b) cones  c) circles  d) right triangles  e) spheres

6. An equation for the tangent plane to the surface \( z = e^{3x} + \sin(y) \) at the point \((0,0,1)\) is:

a) \( z = 3x + 2 \)  b) \( 3x + y + z = 1 \)

\( c) \quad z = 1 + 3x + y \)  \( d) \quad x - y + z = 0 \)

\( e) \quad z = y \)

7. The maximum directional derivative for the function \( f(x, y) = xy^2 \) at the point \((2,3)\) is:

a) \( \frac{288}{5} \)  b) \( 1 \)

\( c) \quad 6 \)  \( d) \quad 12\sqrt{3} \)

\( e) \quad 15 \)

8. When we reverse the order of integration for the iterated integral \( \int_0^1 \int_0^{e^x} dy \, dx \), we get:

a) \( \int_0^1 \int_0^{e^x} \, dx \, dy \)

\( b) \int_0^1 \int_0^{\ln(x)} \, dx \, dy \)

\( c) \int_0^1 \int_0^{e^x} \, dx \, dy \)

\( d) \int_0^1 \int_0^{\ln(x)} \, dx \, dy \)

\( e) \int_0^1 \int_0^{e^x} \, dx \, dy \)
9. When we convert the iterated integral \( \int_0^3 \int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2) \, dy \, dx \)

from rectangular to polar coordinates, we get:

a) \( \int_0^{\pi/2} \int_0^3 \sin(r^2) \, r \, dr \, d\theta \)

b) \( \int_0^{\pi/2} \int_0^3 \sin(r^2) \, r \, dr \, d\theta \)

c) \( \int_0^{\pi/2} \int_0^3 \sin(r^2) \, r \, dr \, d\theta \)

d) \( \int_0^{\pi/2} \int_0^3 \sin(r^2) \, r \, dr \, d\theta \)

e) \( \int_0^{\pi/2} \int_0^3 \sin^2(r) \, r \, dr \, d\theta \)

10. What shape is the solid of integration \( E \) in the triple integral

\[
\iiint_E 1 \, dV = \int_0^{\pi/2} \int_0^{\pi} \int_0^3 \rho^2 \sin(\phi) \, d\rho \, d\phi \, d\theta
\]

11. Suppose that \( \vec{r}'(t) = \langle 2e^{2t}, 3t^2, \cos(t) \rangle \), and \( \vec{r}(0) = \langle 1, 1, 0 \rangle \). Find \( \vec{r}(1) \).

a) \( \langle e^2, 2, \sin(1) \rangle \)

b) \( \langle 2e^2 + 2, 2, 0 \rangle \)

c) \( \langle e^2 + 1, 2, \sin(1) \rangle \)

d) \( \langle 8e^2, 3, \sin(1) \rangle \)

e) \( \langle 8e^2, 4, -\sin(1) \rangle \)

12. Use Green's Theorem to evaluate the line integral

\[
\oint_C 4y \, dx + (5x + y) \, dy
\]

where \( C \) is the closed curve boundary of the triangle below, oriented counterclockwise.

a) 0  
b) 1  
c) 2  
d) 3  
e) 6
13. The length of the curve \( \mathbf{r}(t) = \langle t, t^2, t^3 \rangle \) between the points where \( t = -1 \) and \( t = 1 \) is closest to:

a) 1.3  
b) 2.7  
c) 3.4  
d) 3.7  
e) 4.0

14. Use the Divergence Thm \( \iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div} \mathbf{F} \ dV \) to calculate the flux \( \iint_S \mathbf{F} \cdot d\mathbf{S} \) if \( \mathbf{F}(x,y,z) = \langle 5x + y, z - y, x + y \rangle \) and \( S \) is the boundary surface (6 faces oriented outward) for the box \( E \) shown on the right.

a) 0  
b) \( \pi \)  
c) \( \pi^2 \)  
d) 6  
e) 24

15. The path on the right is the graph of what function?

a) \( z = \cos(x) + \sin(y) \) for \( 0 \leq x \leq 6\pi \) and \( 0 \leq y \leq 6\pi \)  
b) \( \mathbf{r}(t) = \langle \cos(t), \sin(t), t \rangle \) for \( 0 \leq t \leq 6\pi \)  
c) \( \mathbf{r}(u,v) = \langle \cos(u), \sin(u), v \rangle \) for \( 0 \leq u \leq 6\pi \) and \( 0 \leq v \leq 1 \)  
d) \( \mathbf{r}(t) = \langle \sin(t), t, \cos(t) \rangle \) for \( 0 \leq t \leq 6\pi \)  
e) none of a – d

16. The function \( f(s,t) \) is given by the table on the right. \( \frac{\partial f}{\partial s}(1,2) \) is approximately:

<table>
<thead>
<tr>
<th>( s )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>11</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

a) -3  
b) 3  
c) -2  
d) 2  
e) 1
17. Let \( z = x^3 + y^2 \), where \( x = e^t \) and \( y \) is a function of \( t \) graphed on the right.

Use the chain rule to approximate \( \frac{dz}{dt} \) at \( t = 0 \).

a) 1  

b) 4  

c) 0  

d) 7  
e) 12

18. The vector field \( \vec{F} \) is shown in the xy-plane and looks the same in all other horizontal planes.

What statements are true for \( \vec{F} \) at the point labeled P?

a) \( \text{div} \vec{F} < 0, \text{curl} \vec{F} \neq 0 \)  

b) \( \text{div} \vec{F} = 0, \text{curl} \vec{F} = 0 \)  

c) \( \text{div} \vec{F} > 0, \text{curl} \vec{F} = 0 \)  

d) \( \text{div} \vec{F} = 0, \text{curl} \vec{F} \neq 0 \)  
e) \( \text{div} \vec{F} > 0, \text{curl} \vec{F} \neq 0 \)

19. Approximate \( \iiint f(x, y) \, dx \, dy \) by subdividing the region into four equal squares (i.e., \( n = m = 2 \)) and using midpoints to compute a Riemann Sum.

a) 16  

b) 76  

c) 100  

d) 224  
e) 304

20. Evaluate the line integral \( \int_C (\nabla f) \cdot d\vec{r} \), where \( C \) is the curve given parametrically by \( x = t^2 + t \); \( y = 4t - 2 \);

where \( 1 \leq t \leq 2 \), and the level curves of \( f \) are shown above.

a) 0  

b) 1  

c) 3  

d) 4  
e) 5
PART TWO. Longer Answers (50%). These are not multiple choice. Again, SHOW ALL YOUR WORK ON THESE TEST PAGES. CALCULATORS PERMITTED FOR ALL PROBLEMS EXCEPT 27, 28, 29, 30.

21. Let $P(1,0,1), Q(2,3,4)$, and $R(2,2,2)$ be three points in space.

a) Find the angle $\theta$ between the vectors $\overrightarrow{PR}$ and $\overrightarrow{PQ}$.

b) Find an equation for the plane going through the three points.

c) Find parametric equations for the line going through $P$ and $Q$.

22. For the function $f(x,y) = x^3y + 12x^2 - 8y$,

a) find the critical points.

b) For each critical point, determine whether it is a local maximum, a local minimum, or a saddle point. Use the second derivative test with $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$. 
23. Sketch separate graphs in $\mathbb{R}^3$, 3-d space, represented by each of the following equations:

a) $x + 2y + 3z = 6$

b) $y = x$

c) $z = x^2 + y^2$

d) Sketch the solid in 3-d space whose cylindrical coordinates satisfy the inequalities:
   
   $2 \leq r \leq 4$, $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$, $0 \leq z \leq 2$. 

24. A soccer player kicks a ball from the ground toward the center of an empty goal (no goalkeeper). The ball leaves the ground going 50 ft/sec making an angle of 60° with the ground. If he takes the shot from 60 feet in front of the goal and the top of the goal is 8 ft high, will the ball go into the goal in the air? (Ignore air resistance.) Show the acceleration of the ball $\ddot{r}(t)$, its velocity $\dot{r}(t)$, its position $\mathbf{r}(t)$, and solve the problem.
25. Consider the function \( f(x, y) = y - \sqrt{x} \).

a) Sketch the level (contour) curves where \( f(x, y) = 0 \); \( f(x, y) = 1 \); and \( f(x, y) = 2 \).

b) Find \( \nabla f(1,1) \), (gradient of \( f \) at \( (1,1) \)), and draw it on your graph from part a).

c) Find the directional derivative of \( f \) at \( (1,1) \) in the direction \( 4\hat{i} + 3\hat{j} \).

26. a) If \( f(x,y,z) \) is any scalar field (with continuous second partial derivatives), prove that 
\[ \text{curl} \left( \nabla f \right) = < 0,0,0 > \] (i.e., \( \nabla \times \nabla f = < 0,0,0 > \)).

b) Find a potential function \( f(x,y,z) \) for the vector field \( \vec{F}(x,y,z) = < y, x + z, y + 2z > \).
27. Use double integrals to find \( \bar{x} \), the x-coordinate for the center of mass, for the thin triangular plate with vertices \((0,0), (1,0), \) and \((1,1)\), if the density of the plate (mass/area) is given by \( \rho(x,y) = y \).

28. a) Evaluate the triple integral \( \iiint_{0}^{3} \iiint_{0}^{2} \iiint_{0}^{1} 8xyz \, dz \, dy \, dx \).

b) Evaluate a triple integral to find the mass of the right circular cylinder on the right if it has density \( \rho(x,y,z) = x^2 + y^2 \) (mass/vol).
29. a) Evaluate the line integral \( \int_C \vec{F} \cdot d\vec{r} \), where \( \vec{F}(x, y, z) = \langle z, x, y \rangle \) and \( C \) is the line from \((1,1,1)\) to \((3,4,5)\).

b) Is the value for the line integral in part a) the same for any path from \((1,1,1)\) to \((3,4,5)\)? Explain.

30. Evaluate a surface integral to find the flux, \( \iint_S \vec{F} \cdot d\vec{S} \),

where \( \vec{F}(x, y, z) = \langle x, y, z \rangle \) and where \( S \) is the portion of the surface \( z = 9 - x^2 \) over the rectangular region \( R = [0,2] \times [0,3] \) oriented upward. (See the figure on the right.)