NAME:________________________________ ALPHA NUM:__________
INSTRUCTOR: _________________________ SECTION: _____________

CALCULUS III (SM221,SM221P) **FINAL EXAMINATION**

1330-1630 14 Dec 2010 SHOW ALL WORK IN THIS TEST PACKAGE

Tear off the last sheet of this exam. It contains problems 27, 28, 29, 30. Do these problems first WITHOUT YOUR CALCULATOR. When you are finished with these 4 problems, return the sheet to your instructor, take out your calculator, and complete the rest of the exam. For all of the exam you should have as a formula sheet a copy of page 1105 from the text.

PART ONE: MULTIPLE CHOICE (50%). The first 20 problems are multiple-choice. Fill in the best answer on your Scantron bubble sheet. Write your name, alpha number, instructor, and section number on this test and your bubble sheet and bubble in your alpha number. There is no extra penalty for wrong answers on the multiple choice. Show all your scratch work on this test.

CALCULATORS ARE PERMITTED FOR THIS MULTIPLE CHOICE SECTION.

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1. For $g(x,y,z) = 3x^2y - y^2z + z$, $g_y(1,2,3) =$

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2. Which equation below describes a line **PARALLEL** to the plane with equation $3x - 2y + z = 7$?

a. $\mathbf{r}(t) = \langle 1+3t, -2t, 2+t \rangle$
b. $\mathbf{r}(t) = \langle 1+2t, 3t, 2 \rangle$
c. $\mathbf{r}(t) = \langle 1+t, t, 2+t \rangle$
d. $\mathbf{r}(t) = \langle 1+t/3, -t/2, 2+t \rangle$
e. $\mathbf{r}(t) = \langle 7+t, 7+t, 7+t \rangle$

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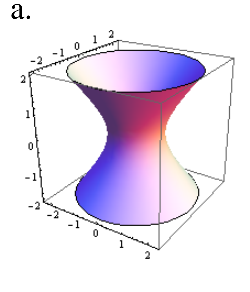
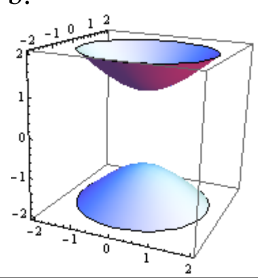
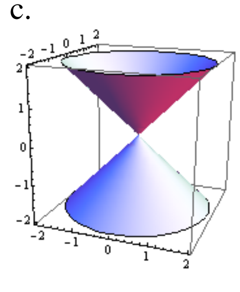
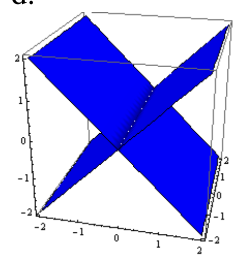
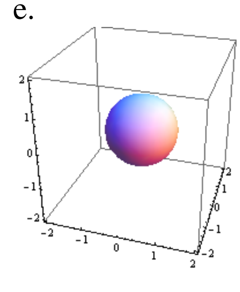
3. Match the equation $x^2 + y^2 - z^2 = 1$ with the appropriate graph below:

a.


b.


c.


d.


e.


4. Which of the following is a tangent vector to the curve given by $\mathbf{r}(t) = \ln(t)\mathbf{i} - 2\sqrt{t}\mathbf{j} + 3\mathbf{k}$ at the point $(0, -2, 3)$?

a. $\mathbf{i} - \mathbf{j}$ b. $-2\mathbf{j} + 3\mathbf{k}$ c. $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ d. $-\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$ e. does not exist

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5. Find, to 3 digit accuracy, the length of the curve in space (a part of a parabola) given by $\mathbf{r}(t) = \langle t, t^2, 3\rangle$ that goes from $(0,0,3)$ to $(1,1,3)$.

a. 1.00 b. 1.40 c. 1.48 d. 1.51 e. 2.33

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6. Which correctly describes both the domain (the biggest set on which the formula makes sense) and the range of the function given by $f(x, y) = \sqrt{y - x + 2}$?

a. The domain is a region on and **above** a line and the range is the interval $[0, \infty)$.
b. The domain is a region on and **below** a line and the range is the interval $(-\infty, \infty)$.
c. The domain is a region on and **below** a line and the range is the interval $(-\infty, 0)$.
d. The domain is a region on and **below** a line and the range is the interval $[0, \infty)$.
e. The domain is a region on and **above** a line and the range is the interval $(-\infty, \infty)$.

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7. For $h(x, y) = \sin(2x) + \cos(3x) + e^{x+y}$, the linear approximation at $(0,0)$ is given by $L(x, y) =$

a. $2 - 3x + y$ b. $3x + y$ c. $\langle 3,1\rangle$
d. $2 + 3x + y$ e. $2 + (2\cos(2x) - 3\sin(3x) + e^{x+y})x + (e^{x+y})y$

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8. If the gradient of f at the origin is given by $\nabla f(0,0) = \langle -2, 3\rangle$, then the fastest rate of increase of f at the origin is:

a. 1 b. $\sqrt{5}$ c. $\sqrt{13}$ d. 5 e. 13

9. A contour map is shown for a function f on the square $R = [0,4] \times [0,4]$. Use the Midpoint Rule with $m = n = 2$ to approximate $\iint_R f(x,y)dA$.

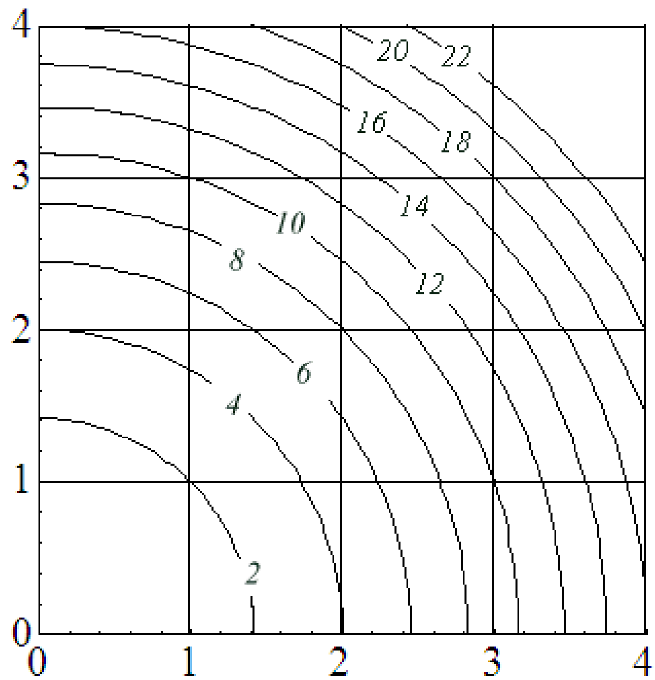
a. 26

b. 40

c. 80

d. 120

e. 160



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10. Which of the following correctly reverses the order of integration for $\int_0^4 \int_{\sqrt{y}}^2 f(x,y)dxdy$?

a. $\int_0^2 \int_0^{x^2} f(x,y)dydx$ b. $\int_0^4 \int_0^{x^2} f(x,y)dydx$ c. $\int_{\sqrt{y}}^2 \int_0^4 f(x,y)dydx$

d. $\int_0^2 \int_0^{\sqrt{x}} f(x,y)dydx$ e. $\int_0^2 \int_{x^2}^4 f(x,y)dydx$

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11. Find the mass of a lamina that lies between the circles centered at the origin given by $r = 1$ and $r = 2$ if the density equals the reciprocal of distance from the origin.

a. π b. 2π c. 3π d. 4π e. $2\pi \ln(2)$

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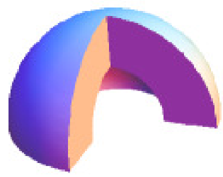
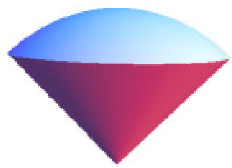
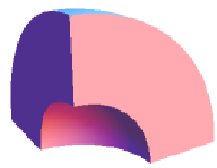
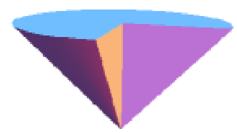
12. Evaluate the triple integral: $\iiint_E x\, dV$ where E is the region bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $3x + 2y + z = 6$. (Suggestion: set it up and then use your calculator.)

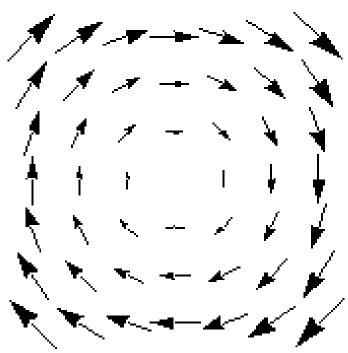
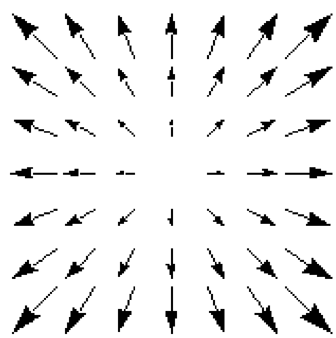
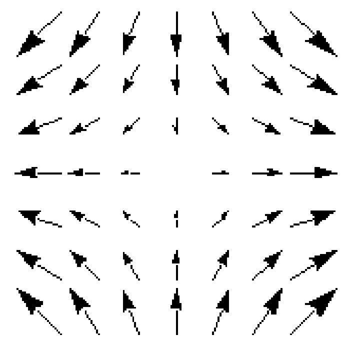
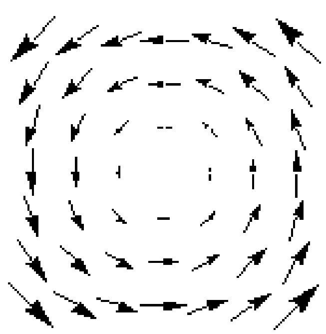
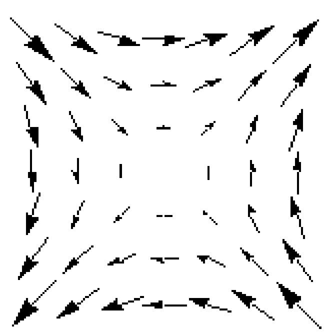
a. 3 b. 4 c. 5 d. 6 e. 7

13. Which figure (perhaps rotated) shows the solid described by spherical coordinates

$$0 \le \rho \le 2,\ 0 \le \phi \le \pi/4,\ 0 \le \theta \le 2\pi$$

a.	b.	c.	d.	e.
				

14. Match the vector field, $\mathbf{F}(x,y) = \langle y, x\rangle$. (Each picture is in the square $[-1,1] \times [-1,1]$, vector lengths may be scaled.)

a.	b.	c.
		
d.	e.	
		

15. Compute $\int_C y\,dx + (x+y)\,dy$ for C the curve given by $x = t+1, y = 2t, 0 \le t \le 1$.

a. 0 b. 5/2 c. 3 d. 7/2 e. 6

16. Find $\int_C \nabla f \cdot d\mathbf{r}$, where C is the curve given by $\mathbf{r}(t) = \langle t, t^2 + 1 \rangle$, $0 \leq t \leq 1$, and f is a function with a continuous gradient and with values $f(x, y)$ given in the table.

$x \setminus y$	0	1	2
0	1	2	3
1	5	8	13
2	19	25	32

a. -4 b. 0 c. 1 d. 4 e. 11

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17. Find $\int_C y dx + 4x dy$ where C is a circle described counterclockwise enclosing an area of 4π. (Hint: Green's Theorem – formula given on formula sheet)

a. 2π b. 4π c. 6π d. 12π e. 16π

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18. For **F** a vector field, which of the following expressions is meaningful AND is a vector field?

a. $\text{div}(\text{curl}(\mathbf{F}))$ b. $\text{grad}(\text{curl}(\mathbf{F}))$ c. $\text{curl}(\text{div}(\mathbf{F}))$ d. $\text{curl}(\text{curl}(\mathbf{F}))$ e. $\text{curl}(\text{grad}(\mathbf{F}))$

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19. Which of the following gives a parameterization of a cylindrical surface?

a. $\mathbf{r}(\theta, z) = \langle 2\cos(\theta), 2\sin(\theta), z \rangle$
b. $\mathbf{r}(\theta, z) = \langle z\cos(\theta), z\sin(\theta), z \rangle$
c. $\mathbf{r}(\theta, z) = \langle \theta, z, \theta^2 + z^2 \rangle$
d. $\mathbf{r}(\theta) = \langle 3\cos(\theta), 3\sin(\theta), 3 \rangle$
e. $\mathbf{r}(\theta, z) = \langle \sin(z)\cos(\theta), \sin(z)\sin(\theta), \cos(z) \rangle$

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20. Calculate the flux of **F** through surface S, in other words $\iint_S \mathbf{F} \cdot d\mathbf{S}$, for $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and S the surface of the cube with $0 \leq x \leq 2$, $0 \leq y \leq 2$, and $0 \leq z \leq 2$ and the positive (outward) orientation.

a. 0 b. 3 c. 6 d. 12 e. 24

PART TWO: FREE RESPONSE (50%). The remaining problems are not multiple choice. Answer them on this test paper in the blank space provided. Show the details of your work and box your answers.

21. A projectile is fired with a speed of 500 ft/s at an angle of 45° above horizontal from a hill 300 ft above sea level. Does it pass over a target that is at sea level, 9000 ft away (measured horizontally)? Justify. (Ignore air resistance and use $32\ \text{ft/s}^2$ for the magnitude of acceleration.)

22. A glass aquarium is to be a box without a lid holding a volume of $2\ \text{m}^3$. Find the dimensions that minimize the amount of glass used.

23. a. Describe in words the surface with cylindrical coordinate equation $\theta = \pi/2$.

b. Sketch the one solid described by all the following cylindrical coordinate inequalities:

$$0 \leq r \leq 3, \qquad \frac{\pi}{2} \leq \theta \leq \pi, \quad 0 \leq z \leq 1$$

24. Derive, writing down the appropriate iterated integrals, the volume contained by a sphere of radius a using:

a. Cylindrical coordinates

b. Spherical coordinates

25. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where S is the helicoid with vector equation $\mathbf{r}(u, v) = u\cos(v)\,\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}$ for $0 \le u \le 1, 0 \le v \le \pi$ with upward orientation, and $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z^3\mathbf{k}$. In other words, find the flux of $\mathbf{F}$ across S.

26. Use Stokes' Theorem to evaluate $\iint_S \text{curl}\,\mathbf{F} \cdot d\mathbf{S}$ where S is the part of the circular paraboloid (bowl) $z = x^2 + y^2$ below $z = 4$ oriented upward and $\mathbf{F}(x, y, z) = 2yz\mathbf{i} + x^4\mathbf{k}$.

CALCULATORS ARE **NOT** PERMITTED FOR PROBLEMS 27, 28, 29, 30.

27. a. Use the Chain Rule (show your work, leave your answer in terms of $x, y, s,$ and t) to find $\partial z/\partial s$ and $\partial z/\partial t$ for

$$z = x^2y^3, x = s\cos(t), y = e^{2s+t}.$$

b. Find the rate of change (the directional derivative) of the function $f(x, y) = x^2y$ at the point (3,4) toward the origin.

28. a. Calculate the iterated integral: $\int_0^2 \int_0^1 \int_0^{4-2x-y} dzdxdy$.

b. Sketch the solid region of integration for part a.

29. a. Find a function f such that $\mathbf{F} = \nabla f$ for
$\mathbf{F}(x, y, z) = (3yz + 4x)\mathbf{i} + (3xz - 4z)\mathbf{j} + (3xy - 4y)\mathbf{k}$.

b. Use the result in part a to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for C given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle, 0 \leq t \leq 1$.

30. Show that the curl of a conservative vector field is zero. More precisely, use the definitions of curl and gradient to prove that for f a function of three variables with continuous second order partial derivatives, $\text{curl}(\nabla f) = \mathbf{0}$.