1. For $g(x,y,z) = 3x^2y - y^2z + z$, $g_y(1,2,3) =$

2. Which equation below describes a line PARALLEL to the plane with equation $3x - 2y + z = 7$?
   a. $\mathbf{r}(t) = (1 + 3t, -2t, 2 + t)$
   b. $\mathbf{r}(t) = (1 + 2t, 3t, 2)$
   c. $\mathbf{r}(t) = (1 + t, t, 2 + t)$
   d. $\mathbf{r}(t) = (1 + t/3, -t/2, 2 + t)$
   e. $\mathbf{r}(t) = (7 + t, 7 + t, 7 + t)$

3. Match the equation $x^2 + y^2 - z^2 = 1$ with the appropriate graph below:
4. Which of the following is a tangent vector to the curve given by $r(t) = \ln(t)i - 2\sqrt{t}j + 3k$ at the point $(0, -2, 3)$?
   a. $i - j$   b. $-2j + 3k$   c. $i - 2j + 3k$   d. $-\frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}k$   e. does not exist

5. Find, to 3 digit accuracy, the length of the curve in space (a part of a parabola) given by $r(t) = (t, t^2, 3)$ that goes from $(0,0,3)$ to $(1,1,3)$.
   a. 1.00   b. 1.40   c. 1.48   d. 1.51   e. 2.33

6. Which correctly describes both the domain (the biggest set on which the formula makes sense) and the range of the function given by $f(x, y) = \sqrt{y - x + 2}$?
   a. The domain is a region on and above a line and the range is the interval $[0, \infty)$.
   b. The domain is a region on and below a line and the range is the interval $(-\infty, \infty)$.
   c. The domain is a region on and below a line and the range is the interval $(-\infty, 0)$.
   d. The domain is a region on and above a line and the range is the interval $[0, \infty)$.
   e. The domain is a region on and above a line and the range is the interval $(-\infty, \infty)$.

7. For $h(x, y) = \sin(2x) + \cos(3x) + e^{x+y}$, the linear approximation at $(0,0)$ is given by $L(x,y) =$
   a. $2 - 3x + y$   b. $3x + y$   c. $(3,1)$
   d. $2 + 3x + y$   e. $2 + (2\cos(2x) - 3\sin(3x) + e^{x+y})x + (e^{x+y})y$

8. If the gradient of $f$ at the origin is given by $\nabla f(0,0) = (-2, 3)$, then the fastest rate of increase of $f$ at the origin is:
   a. 1   b. $\sqrt{5}$   c. $\sqrt{13}$   d. 5   e. 13
9. A contour map is shown for a function \( f \) on the square \( R = [0,4] \times [0,4] \). Use the Midpoint Rule with \( m = n = 2 \) to approximate \( \iint_R f(x,y) \, dA \).

\( R = [0,4] \times [0,4] \)

- a. 26
- b. 40
- c. 80
- d. 120
- e. 160

10. Which of the following correctly reverses the order of integration for \( \int_0^4 \int_0^2 f(x,y) \, dx \, dy \)?

\[ f(x,y) = \frac{1}{\sqrt{x^2 + y^2}} \]

- a. \( \int_0^2 \int_0^{x^2} f(x,y) \, dy \, dx \)
- b. \( \int_0^2 \int_0^{x^2} f(x,y) \, dy \, dx \)
- c. \( \int_0^{\sqrt{2}} \int_0^4 f(x,y) \, dy \, dx \)
- d. \( \int_0^2 \int_0^{\sqrt{x^2 + y^2}} f(x,y) \, dy \, dx \)
- e. \( \int_0^2 \int_0^{x^2} f(x,y) \, dy \, dx \)

11. Find the mass of a lamina that lies between the circles centered at the origin given by \( r = 1 \) and \( r = 2 \) if the density equals the reciprocal of distance from the origin.

\[ \rho(r) = \frac{1}{r} \]

- a. \( \pi \)
- b. \( 2\pi \)
- c. \( 3\pi \)
- d. \( 4\pi \)
- e. \( 2\pi \ln(2) \)

12. Evaluate the triple integral: \( \iiint_E x \, dV \) where \( E \) is the region bounded by the planes \( x = 0, y = 0, z = 0, \) and \( 3x + 2y + z = 6 \). (Suggestion: set it up and then use your calculator.)

\[ 3x + 2y + z = 6 \]

- a. 3
- b. 4
- c. 5
- d. 6
- e. 7
13. Which figure (perhaps rotated) shows the solid described by spherical coordinates
\[\rho \leq 2, \ 0 \leq \phi \leq \pi/4, \ 0 \leq \theta \leq 2\pi\]

a. ![Image a]

b. ![Image b]

c. ![Image c]

d. ![Image d]

e. ![Image e]

14. Match the vector field, \( \mathbf{F}(x, y) = (y, x) \). (Each picture is in the square \([-1,1] \times [-1,1] \),
vector lengths may be scaled.)

a. ![Image a]

b. ![Image b]

c. ![Image c]

d. ![Image d]

e. ![Image e]

15. Compute \( \int_C y\,dx + (x + y)\,dy \) for \( C \) the curve given by \( x = t + 1, y = 2t, 0 \leq t \leq 1 \).

a. 0  b. 5/2  c. 3  d. 7/2  e. 6
16. Find \( \int_C \nabla f \cdot dr \), where \( C \) is the curve given by \( \mathbf{r}(t) = (t, t^2 + 1), 0 \leq t \leq 1 \), and \( f \) is a function with a continuous gradient and with values \( f(x,y) \) given in the table.

\[
\begin{array}{c|ccc}
  x & y & 0 & 1 & 2 \\
\hline
  0 & 1 & 2 & 3 \\
  1 & 5 & 8 & 13 \\
  2 & 19 & 25 & 32 \\
\end{array}
\]

a. \(-4\)  b. \(0\)  c. \(1\)  d. \(4\)  e. \(11\)

17. Find \( \int_C ydx + 4xdy \) where \( C \) is a circle described counterclockwise enclosing an area of \( 4\pi \). (Hint: Green’s Theorem – formula given on formula sheet)

a. \(2\pi\)  b. \(4\pi\)  c. \(6\pi\)  d. \(12\pi\)  e. \(16\pi\)

18. For \( \mathbf{F} \) a vector field, which of the following expressions is meaningful AND is a vector field?

a. \( \text{div}(\text{curl}(\mathbf{F})) \)  b. \( \text{grad}(\text{curl}(\mathbf{F})) \)  c. \( \text{curl}(\text{div}(\mathbf{F})) \)  d. \( \text{curl}(\text{curl}(\mathbf{F})) \)  e. \( \text{curl}(\text{grad}(\mathbf{F})) \) 

19. Which of the following gives a parameterization of a cylindrical surface?

a. \( \mathbf{r}(\theta, z) = (2 \cos(\theta), 2 \sin(\theta), z) \)  
b. \( \mathbf{r}(\theta, z) = (z \cos(\theta), z \sin(\theta), z) \)  
c. \( \mathbf{r}(\theta, z) = (\theta, z, \theta^2 + z^2) \)  
d. \( \mathbf{r}(\theta) = (3 \cos(\theta), 3 \sin(\theta), 3) \)  
e. \( \mathbf{r}(\theta, z) = (\sin(z) \cos(\theta), \sin(z) \sin(\theta), \cos(z)) \)  

20. Calculate the flux of \( \mathbf{F} \) through surface \( S \), in other words \( \iint_S \mathbf{F} \cdot d\mathbf{S} \), for \( \mathbf{F}(x, y, z) = xi + yj + zk \) and \( S \) the surface of the cube with \( 0 \leq x \leq 2 \), \( 0 \leq y \leq 2 \), and \( 0 \leq z \leq 2 \) and the positive (outward) orientation.

a. \(0\)  b. \(3\)  c. \(6\)  d. \(12\)  e. \(24\)
21. A projectile is fired with a speed of 500 ft/s at an angle of 45° above horizontal from a hill 300 ft above sea level. Does it pass over a target that is at sea level, 9000 ft away (measured horizontally)? Justify. (Ignore air resistance and use 32 ft/s² for the magnitude of acceleration.)

22. A glass aquarium is to be a box without a lid holding a volume of 2 m³. Find the dimensions that minimize the amount of glass used.
23. a. Describe in words the surface with cylindrical coordinate equation $\theta = \pi / 2$.

b. Sketch the one solid described by all the following cylindrical coordinate inequalities:

$$0 \leq r \leq 3, \quad \frac{\pi}{2} \leq \theta \leq \pi, \quad 0 \leq z \leq 1$$

24. Derive, writing down the appropriate iterated integrals, the volume contained by a sphere of radius $a$ using:

a. Cylindrical coordinates

b. Spherical coordinates
25. Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $S$ is the helicoid with vector equation $\mathbf{r}(u,v) = u \cos(v) \mathbf{i} + u \sin(v) \mathbf{j} + v \mathbf{k}$ for $0 \leq u \leq 1, 0 \leq v \leq \pi$ with upward orientation, and $\mathbf{F}(x,y,z) = xi + yj + z^3k$. In other words, find the flux of $\mathbf{F}$ across $S$.

26. Use Stokes’ Theorem to evaluate $\iint_S \text{curl} \mathbf{F} \cdot d\mathbf{S}$ where $S$ is the part of the circular paraboloid (bowl) $z = x^2 + y^2$ below $z = 4$ oriented upward and $\mathbf{F}(x,y,z) = 2yz \mathbf{i} + x^4 \mathbf{k}$.
27. a. Use the Chain Rule (show your work, leave your answer in terms of $x, y, s, \text{ and } t$) to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ for 

$$z = x^2y^3, \ x = s \cos(t), \ y = e^{2s+t}.$$ 

b. Find the rate of change (the directional derivative) of the function $f(x,y) = x^2y$ at the point $(3,4)$ toward the origin.

28. a. Calculate the iterated integral: $$\int_0^2 \int_0^1 \int_0^{4-2x-y} dz \, dx \, dy.$$ 

b. Sketch the solid region of integration for part a.
29. a. Find a function $f$ such that $\mathbf{F} = \nabla f$ for
$\mathbf{F}(x, y, z) = (3yz + 4x)\mathbf{i} + (3xz - 4z)\mathbf{j} + (3xy - 4y)\mathbf{k}$.

b. Use the result in part a to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ for $C$ given by $\mathbf{r}(t) = (t, t^2, t^3), 0 \leq t \leq 1$.

30. Show that the curl of a conservative vector field is zero. More precisely, use the definitions of curl and gradient to prove that for $f$ a function of three variables with continuous second order partial derivatives, $\text{curl}(\nabla f) = 0$. 