SM221 Final Exam – May 7, 2012 – Answer Key

Part I

1.	c	6.	c	11.	d	16.	a
2.	a	7.	b	12.	a	17.	d
3.	b	8.	c	13.	b	18.	d
4.	e	9.	c	14.	b	19.	b
5.	b	10.	a	15.	a	20.	b

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<u>Part II</u>

21. See Theorem 15, Section 14.6, p. 939 and Theorem 3, Section 16.5, p. 1092 of text.

<u>Part III</u>

22. $f(x,y) = x^3 - 3xy + y^3$. $f_x = 3x^2 - 3y$, $f_y = -3x + 3y^2$

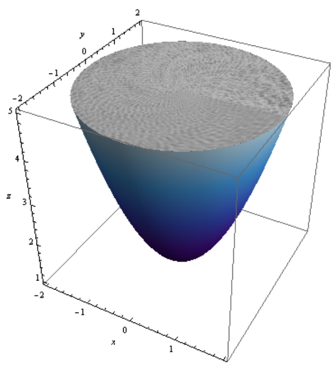
Setting $f_x = 0$ and $f_y = 0$ gives two critical points, $(0,0)$ and $(1,1)$.

The discriminant is $D = f_{xx}f_{yy} - f_{xy}^2 = (6x)(6y) - (-3)^2$.

At $(0,0)$, we have $D = -9 < 0$ so $\boxed{(0,0) \text{ is a saddle point of } f.}$

At $(1,1)$, we have $D = 27 > 0$ and $f_{xx} = 6 > 0$ so $\boxed{f \text{ has a local minimum at } (1,1).}$

23.



$$\boxed{\int_0^{2\pi}\int_0^2\int_{1+r^2}^5 r\,dz\,dr\,d\theta = 8\pi}$$

24. a. $\boxed{f(x,y) = 3xy + 2y^2.}$

b. $f(0,2) - f(1,0) = 8 - 0 = \boxed{8}$

c. $x = 1 - t, y = 2t, 0 \le t \le 1, dx = -dt, dy = 2dt$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_C 3y\,dx + (3x+4y)dy$$
$$= \int_0^1 3(2t)(-1) + \big(3(1-t) + 4(2t)\big)2\,dt = \int_0^1 6 + 4t\,dt = \boxed{8}$$

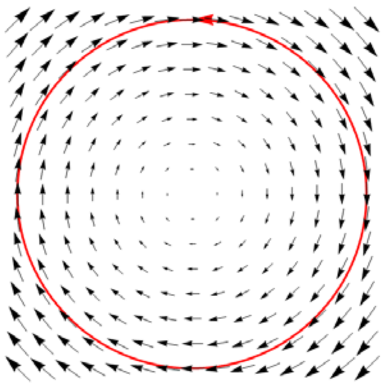
25. $T(x,y,z) = \frac{30}{1+x^2+y^2+z^2}$, $T_x = \frac{-60x}{(1+x^2+y^2+z^2)^2}$, with similar formulas for T_y and T_z.

The rate of change of temperature at the point $(1,1,1)$ in the direction of $\langle 2,-2,1\rangle$ is

$$\nabla T(1,1,1)\cdot\frac{1}{3}\langle 2,-2,1\rangle = \frac{-60}{16}\langle 1,1,1\rangle\cdot\frac{1}{3}\langle 2,-2,1\rangle = \boxed{-\frac{5}{4}}$$

i.e., the temperature is decreasing at 1.25° C per meter.

26. a.



b. $\int_C 2y\,dx - 2x\,dy$

$= \iint_{\text{interior of } C} -4\,dA = \int_0^{2\pi}\int_0^5 -4r\,dr\,d\theta =$

$-4(\text{Area of circle of radius 5}) =$ **-100π.**

c. **No,** $\boldsymbol{F}$ is not conservative because the integral of $\boldsymbol{F}$ around the closed curve C is not zero.

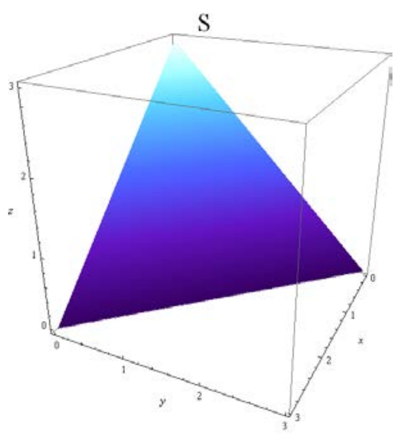
27. The mass of the wire in grams is

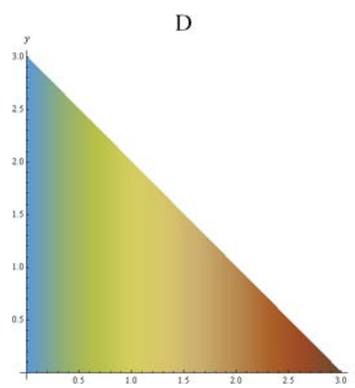
$$m = \int_0^{\pi} 0.1\sqrt{1+t^2}\sqrt{(-2\sin(t))^2 + (2\cos(t))^2 + 1^2}\ dt = \int_0^{\pi} 0.1\sqrt{1+t^2}\sqrt{5}\ dt \cong 1.366.$$

28. The mass of the solid in grams is

$$m = \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^3 2\rho\,\rho^2\sin(\varphi)\ d\rho\,d\varphi\,d\theta = \boxed{81\pi.}$$

29. Let S be the given surface. A parametrization of S is $\boldsymbol{r}(x,y) = \langle x, y, 3-x-y\rangle$, for (x,y) in the region D given by $0 \le x \le 3$ and $0 \le y \le 3-x$.





$\boldsymbol{r}_x = \langle 1,0,-1\rangle,\ \boldsymbol{r}_y = \langle 0,1,-1\rangle,\ \boldsymbol{r}_x \times \boldsymbol{r}_y = \langle 1,1,1\rangle$ (which is upward).

$$\iint_S \boldsymbol{F}\cdot d\boldsymbol{S} = \int_0^3\int_0^{3-x}\langle y, 2x, 3-x-y\rangle\cdot\langle 1,1,1\rangle\,dy\,dx = \int_0^3\int_0^{3-x} 3+x\,dy\,dx = \boxed{18.}$$

30. Let D be the region enclosed by S. By the Divergence Theorem, the charge Q is

$$\varepsilon_0\iiint_D \operatorname{div}\boldsymbol{E}\,dV = \varepsilon_0\int_0^2\int_0^2\int_0^2 2+1+3\,dz\,dy\,dx = \boxed{48\,\varepsilon_0.}$$