Question #1

$$v = \{3, 2, -4\}$$

$$w = \{1, -1, 2\}$$

Part a)

$$2\,v - 3\,w = \{3, 7, -14\}$$

Part b)

$$\text{DotProduct}[v, w] = -7$$

Part c)

$$|v| = \sqrt{3\hat{}2 + 2\hat{}2 + 4\hat{}2} = \sqrt{29}$$

$$|w| = \sqrt{1\hat{}2 + 1\hat{}2 + 2\hat{}2} = \sqrt{6}$$

$$\theta = \text{ArcCos}\left[-7 \Big/ \left(\sqrt{29}\ \sqrt{6}\right)\right] = \text{ArcCos}\left[-\frac{7}{\sqrt{174}}\right] = 2.13019\ \text{rad} = 122\ \text{degree}$$

Part d)

$$\text{proj}_w v = \frac{(v.w)\ w}{(|w|)^2} = \frac{-7}{6}\ \{1, -1, 2\}$$

Part e)

$$\text{DotProduct}[\{x, y, z\}, v] = 0$$

$$3x + 2y - 4z = 0$$

Question #2

Part a)

$$v = \{2 - 1, 4 - 1, 3 - 4\} = \{1, 3, -1\}$$

$$x = 1 + t$$
$$y = 1 + 3t$$
$$z = 4 - t$$

$x = 2 + t$
$y = 4 + 3\ t$
$z = 3 - t$

■ Part b)

v1 = {4 - 1, 1 - 0, 2 - 1} = {3, 1, 1}

v2 = {4 - 0, 1 - 1, 2 - 1} = {4, 0, 1}

CrossProduct[{3, 1, 1}, {4, 0, 1}] = {1, 1, -4}

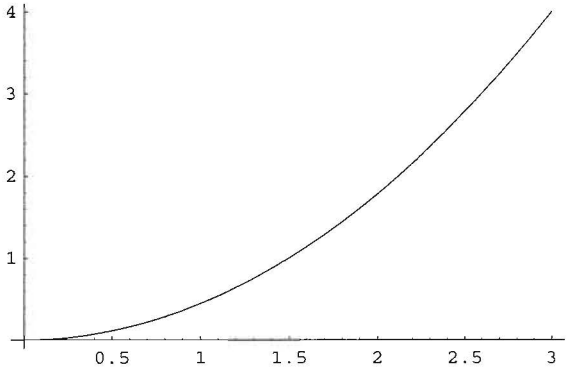
$x + y - 4\ z = -3$ is the plane

■ Question #3

■ Part a)

r[t_] = {3 t, 4 t^2}

$y = 4\ (x / 3)\ ^2$



- Graphics -

■ Part b)

r[1] = {3, 4}

v[t_] = {3, 8 t}

{3, 8 t}

v[1] = {3, 8}

a[t] = {0, 8}

a[1] = {0, 8}

speed = $\sqrt{3^2 + 8^2} = \sqrt{73} = 8.54\ m / s$

■ Part c)

$$x = 3 + 3\,t$$
$$y = 4 + 8\,t$$

■ Part d)

$$\int_1^3 \sqrt{3.\,\hat{}\,2 + (8\,t)\,\hat{}\,2}\,dt = 32.6095$$

■ Question #4

$$r[t\_] = \{40\,\text{Cos}[\pi/6]\,t,\ 40\,\text{Sin}[\pi/6]\,t - 9.8\,t\hat{}\,2/2 + 1\}$$

$$\{20\sqrt{3}\,t,\ 1 + 20\,t - 4.9\,t^2\}$$

$$\text{Solve}\left[20\sqrt{3}\,t == 130,\ t\right]$$

$$\left\{\left\{t \to \frac{13}{2\sqrt{3}}\right\}\right\}$$

$$\text{Solve}\left[20\sqrt{3}\,t == 130.,\ t\right]$$

$$\{\{t \to 3.75278\}\}$$

$$1 + 20\,t - 4.9\,t^2\ /.\ t \to \frac{13}{2\sqrt{3}}$$

$$7.0472$$

It will clear by 3.04 *m*.

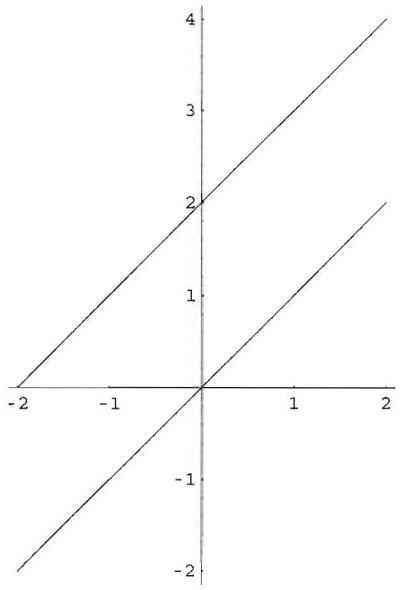
■ Question #5

■ Part a)

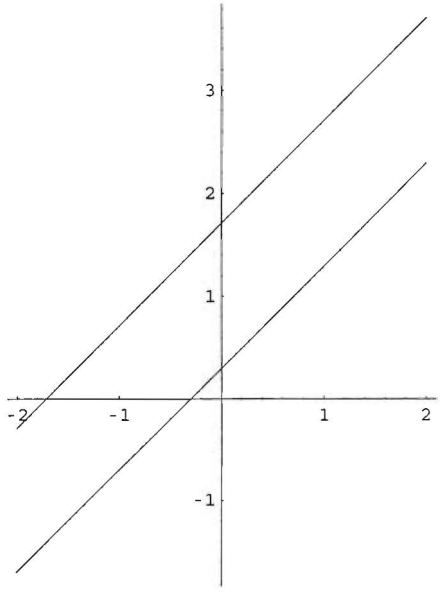
$$f[x\_,\ y\_] = \frac{1}{(1 + x - y)\,\hat{}\,2}$$

Level Curves

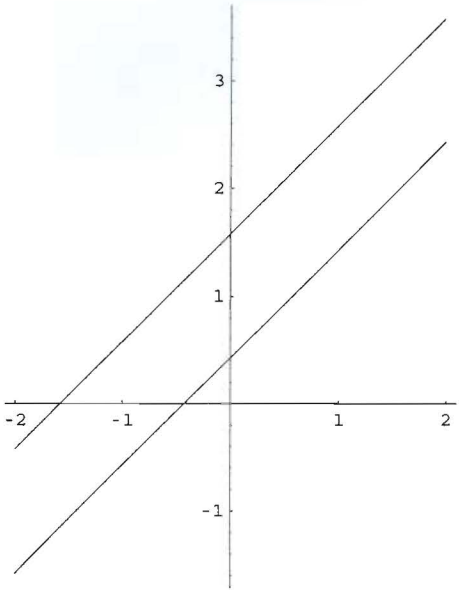
$$a = \text{ImplicitPlot}\left[\frac{1}{(1 + x - y)\,\hat{}\,2} == 1,\ \{x,\ -2,\ 2\}\right]$$

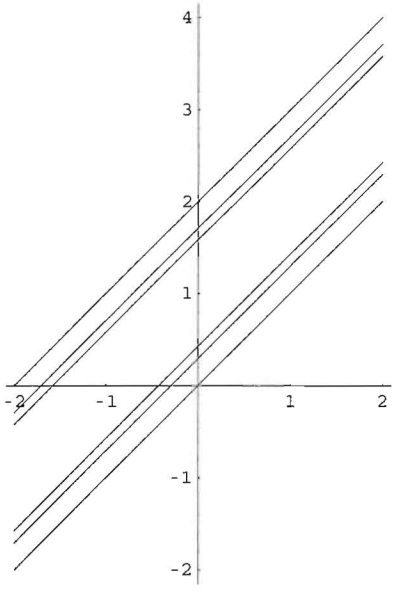



```
b = ImplicitPlot[ 1/(1 + x - y)^2 == 2, {x, -2, 2}]
```




```
c = ImplicitPlot[ 1/(1 + x - y)^2 == 3, {x, -2, 2}]
```





■ Question #7

■ Part a)

i) The amount of rainfall in a storm measured in inches of rain, with 600 miles of the storm is 3 inches of rain.

ii) The rate of change of rainfall with respect the duration of the storm, if the pressure remains the same increases by 0.5 in/hours→rainfall will increase with the duration of the storm.

iii)The rate of change of rainfall with respect the pressure, in a fix duration of time decreases by 0.2 in/cm→ rainfall will decrease with the pressure.

■ Part b)

$$\frac{dr}{ds} = \frac{df}{ds} = \frac{\partial f}{\partial t}\frac{dt}{ds} + \frac{\partial f}{\partial P}\frac{dP}{ds}$$

```
0.5 * 0.01 - 0.2 * (-0.03) = 0.011
```

■ Question #8

```
f[x_, y_] = x^3 y - x y + y^2
```

$$f_x = -y + 3x^2 y$$

$$f_y = -x + x^3 + 2y$$

```
Solve[{-y + 3 x^2 y == 0, -x + x^3 + 2 y == 0}, {x, y}]
```

$$\left\{\{y \to 0, x \to -1\}, \{y \to 0, x \to 0\}, \{y \to 0, x \to 1\}, \left\{y \to -\frac{1}{3\sqrt{3}}, x \to -\frac{1}{\sqrt{3}}\right\}, \left\{y \to \frac{1}{3\sqrt{3}}, x \to \frac{1}{\sqrt{3}}\right\}\right\}$$

$$D = f_{xx} f_{yy} - f_{xy}^2 = -(-1 + 3x^2)^2 + 24 x y$$

$$f_{xx} = 6 x y$$

$$f_{xy} = -1 + 3x^2$$

$$f_{yy} = 2$$

$$D[x_, y_] = 6 x y 2 - (-1 + 3 x^2)\,\hat{}\,2 = -(-1 + 3x^2)^2 + 12 x y$$

```
D2[x_, y_] = -(-1 + 3 x^2)^2 + 12 x y
```

$$-(-1 + 3x^2)^2 + 12 x y$$

$$D[-1, 0] = -4$$

[-1,0] Saddle Point

$$D[0, 0] = -1$$

[0,0] Saddle Point

$$D[1, 0] = -4$$

[1,0] Saddle Point

$$D\left[-\frac{1}{\sqrt{3}}, -\frac{1}{3\sqrt{3}}\right] = \frac{4}{3}$$

$$f_{xx}\left[-\frac{1}{\sqrt{3}}, -\frac{1}{3\sqrt{3}}\right] = 6\left(-\frac{1}{\sqrt{3}}\right) * \left(-\frac{1}{3\sqrt{3}}\right) = \frac{2}{3} > 0$$

$$\left[-\frac{1}{\sqrt{3}}, -\frac{1}{3\sqrt{3}}\right] \text{ Minimum}$$

$$D\left[\frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}\right] = \frac{4}{3}$$

$$f_{xx}\left[\frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}\right] = 6\left(\frac{1}{\sqrt{3}}\right) * \left(\frac{1}{3\sqrt{3}}\right) = \frac{2}{3} > 0$$

$$\left[\frac{1}{\sqrt{3}}, \frac{1}{3\sqrt{3}}\right] \text{ Minimum}$$

■ Question #9

```
S[x_, y_, z_] = x y 5 + 2 y z 3 + 2 x z 2
```

```
g[x, y, z] = x y z = 2
```

```
∂x S[x, y, z] = 5 y + 4 z
```

```
∂y S[x, y, z] = 5 x + 6 z
```

```
∂z S[x, y, z] = 4 x + 6 y
```

```
Solve[
  {5 y + 4 z == λ y z, 5. x + 6 z == λ x z, 4 x + 6 y == λ y x,  x y z == 2}, {x, y, z, λ}]
```

```
{{λ → -3.91487 - 6.78075 i, x → -0.766309 + 1.32729 i,
  y → -0.510873 + 0.884858 i, z → -0.638591 + 1.10607 i},
 {λ → -3.91487 + 6.78075 i, x → -0.766309 - 1.32729 i, y → -0.510873 - 0.884858 i,
  z → -0.638591 - 1.10607 i}, {λ → 7.82974, x → 1.53262, y → 1.02175, z → 1.27718}}
```

```
λ → 7.830, x → 1.532, y → 1.0217, z → 1.277
```

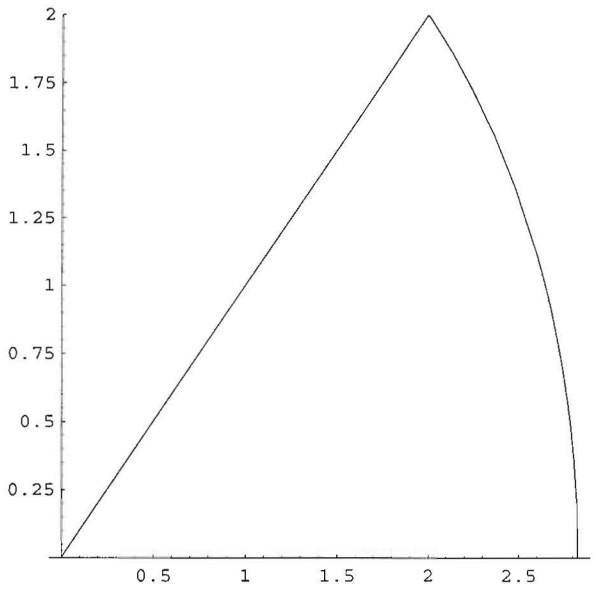
```
x = 15^(2 / 3) 2^(4 / 3.) / 10 = 1.532
```

```
y = 15^(2 / 3) 2^(4 / 3.) / 12 = 1.277
```

```
z = 15^(2 / 3) 2^(4 / 3.) / 15 = 1.021
```

■ Question #10

■ Part a)



■ Part b)

$$\text{Solve}\left[\sqrt{8 - y\hat{}2} == y,\ y\right] \to y = 2$$

$$\int_0^2 \int_0^x \cos\left(x^2 + y^2\right) \mathrm{d}y\, \mathrm{d}x + \int_2^{\sqrt{8}} \int_0^{\sqrt{8-x^2}} \cos\left(x^2 + y^2\right) \mathrm{d}y\, \mathrm{d}x$$

■ Part c)

$$\int_0^{\sqrt{8}} \int_0^{\frac{\pi}{4}} \cos\left(r^2\right) r\, \mathrm{d}\theta\, \mathrm{d}r \text{ or } \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{8}} \cos\left(r^2\right) r\, \mathrm{d}r\, \mathrm{d}\theta$$

■ Part d)

$$\int_0^{\sqrt{8}} \int_0^{\frac{\pi}{4}} \text{Cos}\left[r^2\right] r\, \mathrm{d}\theta\, \mathrm{d}r = \frac{1}{8}\,\pi\,\text{Sin}[8] = \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{8}} \text{Cos}\left[r^2\right] r\, \mathrm{d}r\, \mathrm{d}\theta = 0.3885$$