SM223 Multivariable Calculus FINAL EXAMINATION

Instructions: Ten problems constitute this examination. Answer them in your blue books or on the paper provided by your instructor. Make sure to write your name, alpha number and course section number on each blue book you submit or on *each page* of paper that you submit to your instructor. Return the examination with your solutions. You may use a calculator, such as the Voyage 200 to assist you in solving the problems. Show as much detail in the development of your solutions to receive partial credit for your answers and enclose your answers to problems in a box.

1. Let $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$ and $\mathbf{w} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

a) Find $2\mathbf{v} - 3\mathbf{w}$.
b) Find $\mathbf{v} \bullet \mathbf{w}$.
c) Find the angle between $\mathbf{v}$ and $\mathbf{w}$.
d) Find $\mathbf{proj}_{\mathbf{w}}\mathbf{v}$.
e) Find a vector $\mathbf{z}$ which is perpendicular to $\mathbf{v}$.

2. Resolve the following questions, showing your work.

a) Find the parametric equations of the line passing through the points $(1,1,4)$ and $(2,4,3)$.
b) Find the equation of the plane passing through the points $(1,0,1)$, $(4,1,2)$ and $(0,1,1)$.

3. A silver bug is crawling across a table. The bug's position at time t is given by $\mathbf{r}(t) = 3t\mathbf{i} + 4t^2\mathbf{j}$.

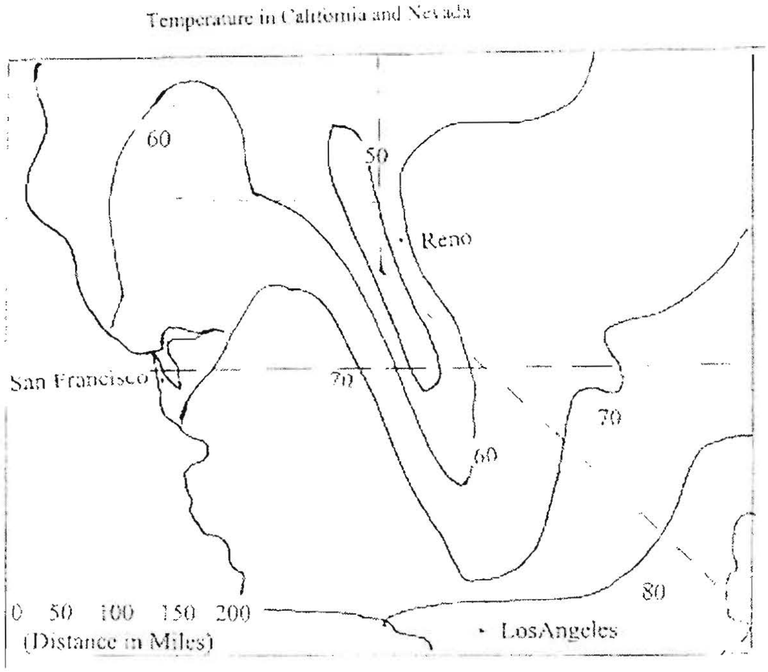
a) Draw the bug's path.
b) Find the bug's position, velocity, acceleration and speed at time $t = 1$.
c) Find the equation of the line tangent to the bug's path at the point $(3,4)$.
d) Find the arclength of the bug's path from $(3,4)$ to $(9,36)$.

4. A baseball is hit 1 meter (m.) above the ground at an angle of 30° above the horizontal and at a velocity of 40 m/s toward right field. The right field fence is 4 m. high and 130 m. from home plate. By how much does the ball clear the right field fence? Explain your calculations.

5. Let $f(x,y) = 1/(1+x-y)^2$

a) Draw three level curves for the function.
b) Draw the graph of the function.

6. The following figure depicts as a contour map the temperature as a function of position in a region of California and Nevada that includes San Francisco, Reno and Los Angeles. Use the map to resolve the questions that follow it.



a) Estimate the gradient vector for the temperature function at San Francisco, at Reno and at Los Angeles.
b) Describe the location of the point at which the temperature is the lowest.
c) Estimate the directional derivative at Reno in the direction due West of Reno.
d) Graph the temperature along the line running due East from San Francisco.

7. The amount of rainfall r in a storm, measured in inches of rain, is a function f of the duration t of the storm, measured in hours, and the pressure P, measured in cm. of mercury.

a) Explain in words what information about the storm each of the following expressions tells you.
   i) $f(6,74)=3$.
   ii) $\partial f/\partial t(6,74)=0.5$.
   iii) $\partial f/\partial p(6,74)=-0.2$.
b) The duration t and the pressure P of the storm are each functions (respectively, h and k) of the width s of the storm, measured in miles. Suppose that $h(600)=6$, $k(600)=74$, $dt/ds(600)=0.01$ and $dP/ds(600)=-0.03$. Using the information in part a), find $dr/ds(600)$.

8. Let $f(x,y)=x^3y-xy+y^2$. Find all critical points and discuss whether each is a local maximum, local minimum or a saddle point.

9. Suppose that you are building a box with an open top. The material for the bottom costs $\$5/\mathrm{m}^2$, the material for the sides costs $\$3/\mathrm{m}^2$, and the material for the front and the back costs $\$2/\mathrm{m}^2$. The volume of the box must be $2\ \mathrm{m}^3$. Use Lagrange Multipliers to find the dimensions of the box which minimizes the cost.

10. Let

$$I=\int_0^2\int_y^{\sqrt{8-y^2}}\cos\left(x^2+y^2\right)dx\,dy$$

a) Draw the region of integration.
b) Rewrite the integral by reversing the order of integration.
c) Write the integral in terms of polar coordinates.
d) Use one of the above representations to evaluate the integral.