SM223 Final
Monday December 20, 2010

MULTIPLE CHOICE SECTION

You may use only one single-sided page of notes for this section. No calculators are allowed.

1. Which plane contains the point $(1, 2, -3)$ and is perpendicular to $\langle 4, 3, 2\rangle$?
a) $x + 2y - 3z = 4$
b) $x + 2y - 3z = 16$
c) $4x + 3y + 2z = 0$
d) $4x + 3y + 2z = 4$
e) $4x + 3y + 2z = 16$

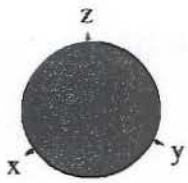
2. What is the cross product $(\mathbf{i} \times \mathbf{j}) \times \mathbf{k}$?
a) $\mathbf{k}$
b) $-\mathbf{k}$
c) $\mathbf{i} + \mathbf{j}$
d) $\vec{0}$
e) $\mathbf{i} + \mathbf{j} + \mathbf{k}$

3. What is the cosine of the angle between $\langle 2, 2, 1\rangle$ and $\langle 0, 1, 0\rangle$?
a) $\frac{2}{9}$
b) $\frac{1}{3}$
c) 0
d) $\frac{\pi}{3}$
e) $\frac{2}{3}$

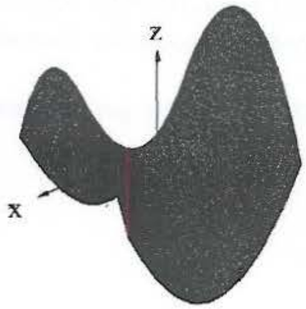
4. What is the magnitude of the vector $\langle 1, -6, 3\rangle + \langle 0, 8, -1\rangle$?
a) 9
b) 3
c) $\langle 1, 2, 2\rangle$
d) $\sqrt{8}$
e) 0

5. Which of the following could represent the plot of the equation $x = y^2 + z^2$?

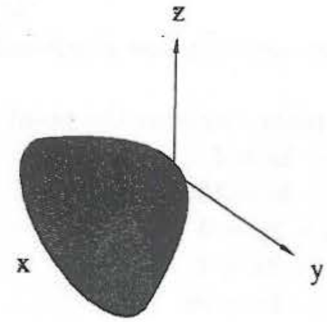
a)



b)

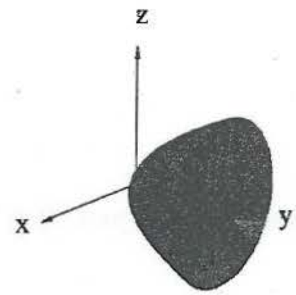


c)

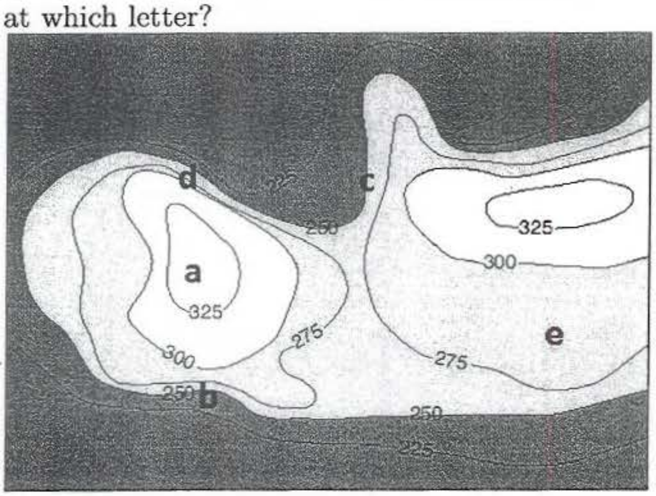


d)

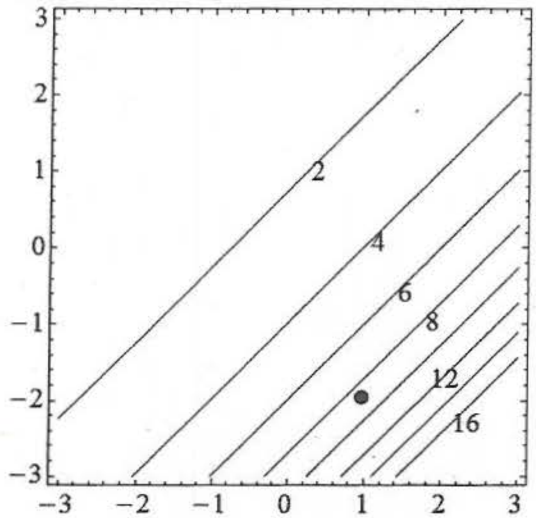
e)



6. Of the five points marked with letters on the contour map below, the largest value of $f_y(x, y)$ is at which letter?



7. Which of the following statements represents the best available information about the function whose contour plot is shown below? The dot is located at (1,-2).

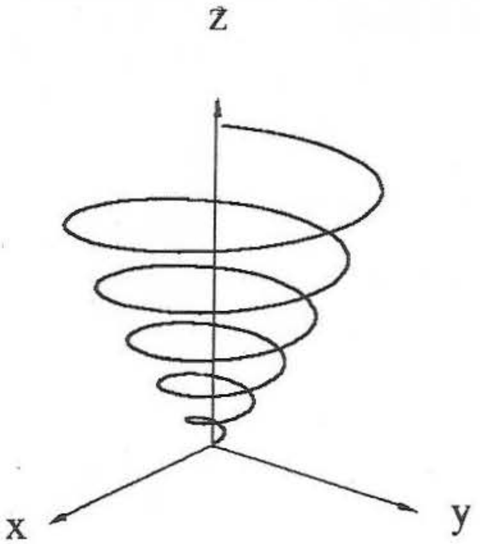


a) $f_x(1,-2) < 0$ and $f_{xx}(1,-2) < 0$
b) $f_x(1,-2) < 0$ and $f_{xx}(1,-2) > 0$
c) $f_x(1,-2) < 0$ and $f_{xx}(1,-2) = 0$
d) $f_x(1,-2) > 0$ and $f_{xx}(1,-2) > 0$
e) $f_x(1,-2) > 0$ and $f_{xx}(1,-2) < 0$

8. If $v(t) = \langle t, t^2, t^3 \rangle$ is the velocity of a flying ladybug at time t in seconds and the ladybug starts at $(1,1,1)$ at time $t = 0$ then where is the ladybug after one second?
a) $(2,3,4)$
b) $(3/2, 4/3, 5/4)$
c) $(1,1,1)$
d) $(1/2, 1/3, 1/4)$
e) $(1,2,3)$

9. Find the length of the curve: $\langle \sin(t),\ \cos(t), t\sqrt{3}\ \rangle$ from t = 0 to t = 10.
a) $10 + 50\sqrt{t}$
b) $\cos(10) + \sin(10) + 10\sqrt{3}$
c) $10 + 10\sqrt{3}$
d) 10
e) 20

10. Which of the below vector functions could have the following plot?



a) $r(t) = \langle \cos(t),\, \sin(t),\, t \rangle$
b) $r(t) = \langle t^2,\, t^2,\, 1/t \rangle$
c) $r(t) = \langle t\cos(t),\, t\sin(t),\, t \rangle$
d) $r(t) = \langle \cos(t),\, \sin(t),\, 1/t \rangle$
e) $r(t) = \langle t,\, t\sin(t),\, t\cos(t) \rangle$

11. Find $f_y(-1, 1)$ if $f(x, y) = (x + 2y)^3$.
a) 8
b) 3
c) -3
d) -6
e) 6

12. Use the table of values for $T(x, y)$ to estimate the partial derivative $T_x(0, 0)$.

$T(x,y)$	$y=0$	$y=2$	$y=4$
$x=0$	5	10	16
$x=3$	7	11	15
$x=6$	11	12	15

a) $\frac{2}{3}$

b) $\frac{5}{3}$

c) $\frac{5}{2}$

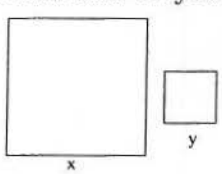
d) 2

e) 3

13. You are hiking on a mountain whose elevation z is given by $z = f(x, y) = 9 - x^2 - 2y$. Assume x-coordinates show east-west and y-coordinates show north-south, as in the familiar compass directions. If you are at the point (1,3,2), which way should you head to ascend the fastest?
a) southeast
b) southwest
c) due east
d) northwest
e) due south

14. The gradient of a function $f(x, y)$ is $\nabla f = \langle \sin(yz), xz\cos(yz), xy\cos(yz)\rangle$. Find the directional derivative at the point (1,3,0) in the direction $u = \langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$.
a) $\langle 0, 0, -1\rangle$
b) $\langle 0, 0, 3\rangle$
c) -1
d) 1
e) -3

15. A wire 40 inches long will be cut to enclose two square-shaped regions. Let the side of one region have length x and the side of the second region have length y. (If an optimal solution has $x = 0$ or $y = 0$, that would correspond to using all the wire to make only one square.) In setting up a maximization problem to maximize the area enclosed in the regions, $f(x, y) = x^2 + y^2$, what constraint do you need?



a) $x^2 + y^2 = 40$
b) $2x + 2y = 40$
c) $4x + 4y = 40$
d) $x + y = 40$
e) $x^2 + y^2 = 1600$

16. What is the shape of the region of integration for this double integral in polar coordinates?
$\int_0^{2\pi} \int_2^3 f(r, \theta) r \, dr \, d\theta$
a) square
b) circle
c) half circle
d) donut (another word for this is annulus)
e) half donut (another word for this is annulus)

17. Suppose X and Y are random variables, where X is the wait time to buy a movie ticket and Y is the wait time to buy a large popcorn. Let X and Y have joint density function $f(x, y) = 0.1e^{-(.5x+.2y)}$ for all $x \geq 0$, $y \geq 0$, and $f(x, y) = 0$ otherwise. Which integral gives the probability that you wait longer than 3 minutes to buy your ticket?

a) $\int_0^\infty \int_0^\infty 0.1e^{-(.5x+.2y)}\, dx\, dy$

b) $\int_0^\infty \int_0^3 0.1e^{-(.5x+.2y)}\, dx\, dy$

c) $\int_0^\infty \int_3^\infty 0.1e^{-(.5x+.2y)}\, dx\, dy$

d) $\int_3^\infty 0.1e^{-(.5x+.2y)}\, dx$

e) $\int_0^3 0.1e^{-(.5x+.2y)}\, dx$

18. Calculate the iterated integral $\int_1^4 \int_1^2 2x\, dy\, dx$

a) 7
b) 0
c) 15
d) 9
e) $\frac{x^3}{3}$

19. Rewrite the following integral in spherical coordinates:

$\int_{-2}^2 \int_0^{\sqrt{(4-y^2)}} \int_{-\sqrt{(4-x^2-y^2)}}^{\sqrt{(4-x^2-y^2)}} \cos(x^2 + y^2 + z^2)\, dz\, dx\, dy$

a) $\int_0^\pi \int_0^\pi \int_0^2 \cos(\rho^2)\rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

b) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^\pi \int_0^2 \cos(\rho^2)\rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

c) $\int_0^\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 \cos(\rho^2)\rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

d) $\int_0^{\frac{\pi}{2}} \int_0^\pi \int_0^2 \cos(\rho^2)\rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

e) $\int_0^\pi \int_0^{\frac{\pi}{2}} \int_0^2 \cos(\rho^2)\rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$

20. Find the distance from the origin to the point with cylindrical coordinates $(r, \theta, z) = (3, \pi, 4)$.

a) 5
b) $3 + \pi + 4$
c) $\sqrt{25 + \pi^2}$
d) $5 + \pi$
e) $7 + \pi$

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FREE RESPONSE SECTION

Write solutions to these problems in your blue books. Be sure to number each solution. You may use your calculator and one single-sided page of notes.

21. a) List three points on the plane $3x - 3y + z = 9$.
b) Find the parametric equations of a line through the points $(1, 2, 3)$ and $(0, -1, -1)$.

22. An athlete throws a shot at an angle of 56° to the horizontal at an initial speed of 43 feet per second. It leaves his hand 7 feet above the ground. Use the gravitational constant $g = -32.2$ feet per second per second.
a) How high is the shot at t=1, 1 second after it leaves his hand?
b) When does the shot land?
c) How far away from the athlete does the shot land?

23. In this problem, we will use a linearization of the function $f(x, y) = x\sqrt[3]{y}$ to approximate the value of: $1.1\sqrt[3]{7.87}$.
For this problem, a convenient point to linearize around is $(1, 8)$. A point is convenient for this purpose if the function and its derivatives are easy to evaluate.
a) Find a linearization $L(x, y)$ around the point $(1, 8)$.
b) Use $L(x, y)$ to approximate $1.1\sqrt[3]{(7.87)}$.
c) Find the absolute error from using linearization to estimate $1.1\sqrt{(7.87)}$. The absolute error is the absolute value of the difference between the correct value and the value you estimate using linearization.

24. Consider a function $f(a, d)$ that predicts the number of widgets that Widget Co. will sell next year if it invests a dollars in advertising and d dollars in development of new products.
a) Describe what $f(2000, 5000) = 12000$ means for the company's widget sales next year.
b) Describe what it means to the company's widget sales next year that $f_d(2000, 5000) = 5$.

25. The trunk of a young redwood tree may be regarded as a cone, and thus its volume is given by

$$V(r, h) = \frac{\pi}{3}r^2h.$$

Currently a redwood tree has height 500 cm and radius 5 cm. The tree is growing at a rate of 12 cm/yr in height, while its radius is increasing at a rate of $\frac{1}{2}$ cm/yr.
a) Use the Chain Rule to write down a general expression for dV/dt.
b) Use the Chain Rule to determine the rate at which the volume of the redwood's trunk is changing. You can leave your answer in terms of π. (Include units.)

26. This problem deals with the function $f(x, y) = xy(12 - 3x - 2y)$.
a) One can show that $f_x = 2y(6 - 3x - y)$. Find an expression for f_y.
b) The function f has four critical points, two of which are $(x, y) = (0, 0)$ and $(x, y) = (4/3, 2)$. List the other two critical points.
c) Classify each of the four critical points as a relative maximum, relative minimum, or saddle point. **Hint.** The function f satisfies

$$f_{xx} = -6y, \qquad f_{yy} = -4x, \qquad f_{xy} = 12 - 6x - 4y.$$

d) A rectangular box is in the first octant (where all coordinates are non-negative). One vertex is at the origin and the opposite vertex at a point (x, y, z) on the plane

$$3x + 2y + z = 12.$$

The edges of the box are parallel to the coordinate axes. Find the maximum volume of the box. **Hint.** How is the volume of the box related to the function $f(x, y)$?

27. When a farmer uses x thousand cubic yards of water per acre and y pounds of fertilizer per acre, the yield for a crop is

$$f(x, y) = 13 + xy \qquad \text{bushels/acre.}$$

Water costs \$3 per thousand cubic yards, and fertilizer costs \$4/pound. The farmer has budgeted \$600/acre. The farmer will use Lagrange multipliers to maximize the yield.
a) The constraint is of the form $g(x, y) = k$. Find the function g and the constant k.
b) List the three equations in three unknowns (x, y, and λ) that arise from the method of Lagrange multipliers.
c) Find the values of x and y that maximize yield.

28. a) Sketch the solid Q bounded by $y + 3z = 3$, $z = 0$, $y = 0$, $x = 0$ and $x = 2$.
b) Set up a triple integral in rectangular coordinates, integrating the function $f(x, y, z) = 1$, to find the volume of the solid Q.
c) The moment of inertia (also called rotational inertia) around the z-axis measures an object's resistance to rotation around the z-axis. The larger an object's rotational inertia, the more force will be required to make it spin. If a solid Q has density $\omega(x, y, z)$, then the moment of inertia I_z can be calculated as: $I_z = \iiint_Q (x^2 + y^2)\omega(x, y, z)\, dV$.

Calculate the rotational inertia of the solid Q described in part (a) of this question, assuming that $\omega(x, y, z) = y^2$

29. In this problem we consider the iterated integral

$$\int_0^2 \int_{x^2}^4 \frac{3}{1 + y^{3/2}}\, dy\, dx\,.$$

a) Sketch the region of integration.
b) Rewrite the integral with the order of integration reversed.
c) Evaluate the integral.