**SM223 Calculus 3 Fall 2014–15 FINAL EXAM ANSWERS**

## Part I Multiple Choice

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | C | D | D | E | A | E | D | B | C | A | D | C | B | B | C | E | E | D | C |

## Part II Long Answer

1. (a) $\mathbf{u} = \frac{1}{3}\langle 2, 2, 1\rangle$ or $\mathbf{u} = -\frac{1}{3}\langle 2, 2, 1\rangle$
   (b) $x = 2t + 1,\ y = 2t + 2,\ z = 1t + 12.$ (Other correct answers exist.)
   (c) $(5, 6, 14)$ at $t = 2$.
   (d) $\text{dist}(P, \text{plane}) = \text{dist}(P, \text{answer to (c)}) = 6$
2. (a) $A = (5, 0, 0),\ B = (0, 2, 0)$
   (b) $\overrightarrow{CA} \cdot \overrightarrow{CB} = \langle 5, 0, -10\rangle \cdot \langle 0, 2, -10\rangle = 100$
   (c) $\angle ACB = \arccos\left(\frac{100}{\sqrt{125}\sqrt{104}}\right) \doteq 0.501$ radians or $28.7°$
   (d) $\overrightarrow{CA} \times \overrightarrow{CB} = \langle 20, 50, 10\rangle$.
   (e) area of $\triangle ABC = \frac{1}{2}|\langle 20, 50, 10\rangle| = 5\sqrt{30} \doteq 27.4$
3. (a) $|\mathbf{v}(2)| = |\langle 5, 0, 12\rangle| = 13$ m/min.
   (b) $x = 5t + 6,\ y = 0t + 7,\ z = 12t + 1.$ (Other correct answers exist.)
   (c) $\mathbf{r}(t) = \langle t^2 + t,\ 0,\ t^3\rangle + \langle 0, 7, -7\rangle$
4. (a) $\mathbf{r}(t) = \big\langle x(t),\ y(t)\big\rangle = \langle 20\sqrt{3}\,t,\ -4.9t^2 + 20t + 50\rangle$
   (b) Solve $y(t) = 0$ to get time of impact $t^* \doteq 5.83$ sec.
   (c) i. $x(t^*) \doteq 202$ m
      ii. $x(t^*)$ is the *range* of the projectile—the distance from the landing point to the base of the cliff.
   (d) 228 m is the (approximate) distance traveled by the projectile on its trajectory through the air from launch to impact, i.e., the arc length.
5. (a) i. $P(3, 60) = 20$ : The manufacturer has a profit of \$20,000 per month if there are 3 hours of radio ads and the key component costs \$60.
      ii. $P_c(3, 60) = -0.3$ : Suppose the manufacturer places 3 hours of radio ads and the key component costs \$60. For every \$1 increase in cost of the key component, the monthly profit decreases by \$300.
   (b) $P_r(3, 60) \doteq \frac{25-15}{5-1} = 2.5.$
   (c) $P(4, 62) \approx 20 + 2.5(4 - 3) - 0.3(62 - 60) = 21.9.$

6. (a) (i) $f(Q) - f(P) = 0$ (ii) $f(P) - f(Q) = 8 - 8 = 0$
   (b) (i) $f_x(P) < 0$ (ii) $f$ decreases in the $x$-direction through $P$.
   (c) (i) $f_{xx}(P) < 0$ (ii) $f$ is concave down in the $x$-direction through $P$.
   (d) (i) $f_{xx}(Q)f_{yy}(Q) - [f_{xy}(Q)]^2 < 0$ (ii) $Q$ is a saddle point.
7. $f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2$.

$$f_x = 6xy - 12x = 6x(y-2) \qquad \text{and} \qquad f_{xx} = 6y - 12.$$

$$f_y = 3y^2 + 3x^2 - 12y, \quad f_{yy} = 6y - 12, \quad f_{xy} = 6x, \quad D = (6y-12)^2 - (6x)^2$$

   (a) $(0, 0)$ is a maximum.
   (b) $(0, 4)$ is a minimum; $(2, 2)$ is a saddle point; $(-2, 2)$ is a saddle point.

8. (a) objective function: $A(x, y) = 4xy$
   (b) constraint: $g(x, y) = 4x + 5y = 120 = k$
   (c) maximum area = 720 ft$^2$ with $x = 15$ and $y = 12$
   The system arising from Lagrange multipliers is:
   $\{4y = \lambda \cdot 4;\ 4x = \lambda \cdot 5;\ 4x + 5y = 120\}$.

9. (a) i. polar: $(r, \theta) = (10, \pi/4)$
   ii. rectangular (Cartesian) $(x, y) = (5\sqrt{2},\ 5\sqrt{2})$
   (b) i. rectangular (Cartesian)

$$\int\int_R f(x, y)\, dA = \int_0^{5\sqrt{2}} \int_x^{\sqrt{10^2 - x^2}} x\sqrt{x^2 + y^2}\, dy\, dx$$

   ii. polar

$$\int\int_R f(x, y)\, dA = \int_{\pi/4}^{\pi/2} \int_0^{10} (r\cos(\theta))\sqrt{r^2}\, r\, dr\, d\theta$$

   (c) $2500 - 1250\sqrt{2}$

10. (a) Sketch omitted. It's the lower half of a sphere of radius 2 at origin.
   (b) i. rectangular (Cartesian). One of several correct answers is

$$\int\int\int_E f(x, y, z)\, dV = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{0} (x^2 + y^2 + z^2)^3\, dz\, dy\, dx$$

   ii. spherical. One of several correct answers is

$$\int\int\int_E f(x, y, z)\, dV = \int_0^{2\pi} \int_{\pi/2}^{\pi} \int_0^2 (\rho^2)^3\, \rho^2 \sin(\phi) d\rho\, d\phi\, d\theta$$

   (c) $1024\pi/9$