Part II Long Answer CALCULATOR ALLOWED

Name: ________________ **Alpha:** __________ **Instructor:**____________

Instructions: A calculator is allowed for Part II of this exam. Write your work directly on this exam. Clearly indicate the places you use a calculator.

1. This problem deals with the point $P = (1, 2, 12)$ and the plane

$$2x + 2y + z = 36.$$

(a) Find a unit vector perpendicular to the plane.

(b) Give parametric equations for the line through P perpendicular to the plane.

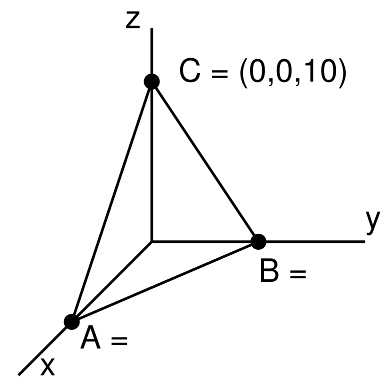
(c) Find the point where the line you found in (a) intersects the plane.

(d) Find the distance between the point P and the plane.

2. The plane

$$2x + 5y + z = 10$$

intersects the x-, y-, and z-axes at the points
A, B, and $C = (0, 0, 10)$, respectively.
The figure is not to scale.



(a) Insert the coordinates for points A and B in the figure.

(b) Compute $\overrightarrow{CA} \cdot \overrightarrow{CB}$.

(c) How big is $\angle ACB$ (in degrees or radians)?

(d) Compute $\overrightarrow{CA} \times \overrightarrow{CB}$.

(e) Find the area of $\triangle ABC$.

3. The velocity of a particle at time t is

$$\mathbf{v}(t) = \mathbf{r}'(t) = \left\langle 2t+1,\ 0,\ 3t^2 \right\rangle,$$

and its position at time $t = 2$ is $\mathbf{r}(2) = \langle 6,\ 7,\ 1 \rangle$. Length is measured in meters, and time is measured in minutes.

(a) Find the speed of the particle at time $t = 2$.

(b) Find parametric equations of the tangent line to the particle's trajectory at $t = 2$.

(c) Find the position $\mathbf{r}(t)$ of the particle at time t.

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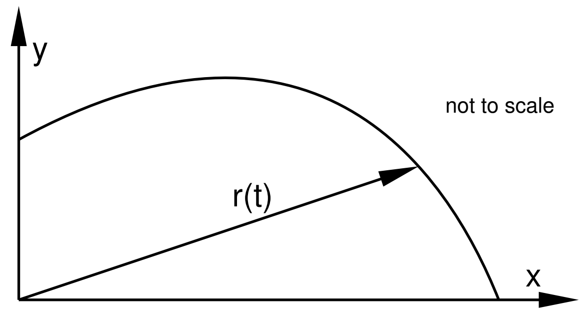
4. A projectile is fired from a cliff of height 50 m with initial speed 40 m/sec at an angle 30° ($\pi/6$ radians) above horizontal.
The position vector for the projectile is

$$\mathbf{r}(t) = \langle x(t),\, y(t) \rangle,$$

as shown. The acceleration due to gravity is $9.8\,\text{m/sec}^2$ directed downward.
Ignore air resistance.



(a) Use the given numbers to find the position vector $\mathbf{r}(t)$.

(b) Compute the projectile's time of impact t^*.

(c) i. Compute $x(t^*)$.

ii. Explain what the number you computed means in words. You can annotate and refer to the diagram.

(d) A calculator gives

$$\int_0^{t^*} |\mathbf{r}'(t)|\, dt = \int_0^{t^*} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \doteq 228.$$

What does this number mean in the context of this problem?

5. The function $P = P(r, c)$ gives the monthly profit (in thousands of dollars) earned by a small manufacturer that places r hours of ads on the radio, and pays a supplier c dollars for a key component in each product. The table gives some values of P.

$P(r,c)$	$c = 50$	$c = 60$	$c = 70$
$r = 1$	19	15	11
$r = 3$	23	20	17
$r = 5$	27	25	23

(a) Explain in words understandable by a 9th grader what the following assertions mean. Do not use words technical words such as "partial" or "derivative." Include units.

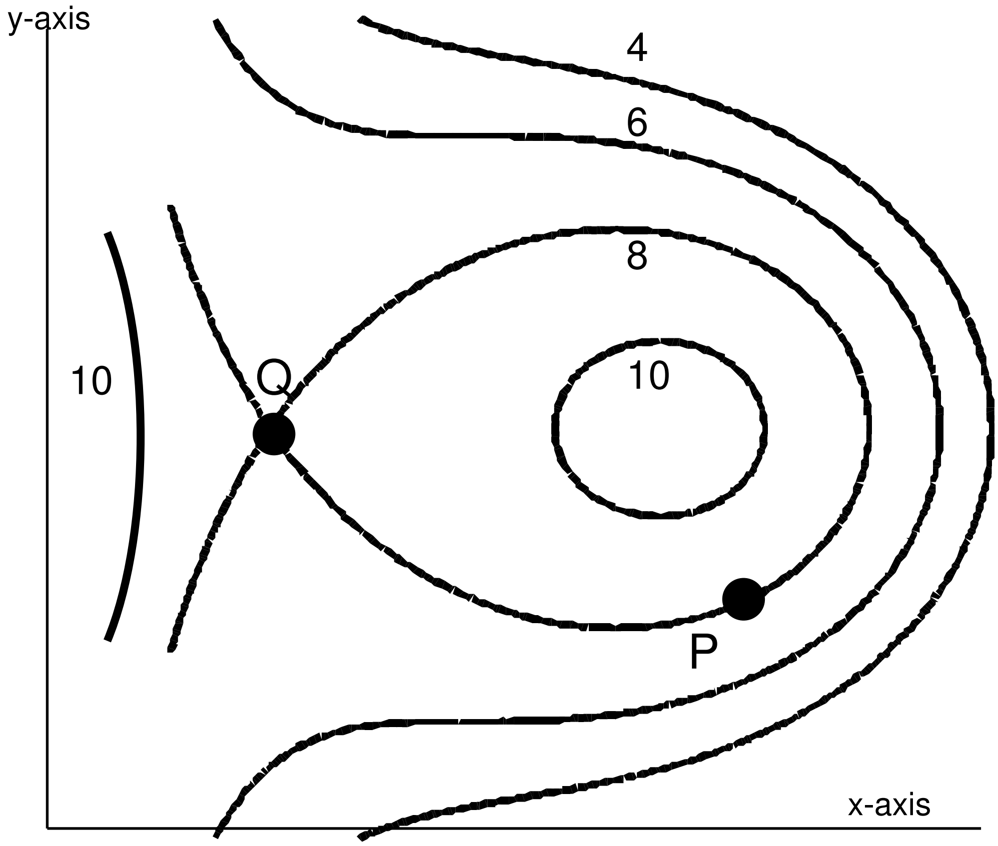
i. $P(3, 60) = 20$

ii. $P_c(3, 60) = -0.3$

(b) Use the table of values to estimate the partial derivative $P_r(3, 60)$.

(c) Use a linear approximation (and the values from (a) and (b)) to estimate $P(4, 62)$.

6. The figure shows the contour diagram for a smooth function $f(x, y)$ with two points labeled $P$and Q. For each expression
(i) state the sign (positive, negative, or zero)
(ii) explain your answer.



(a) $f(Q) - f(P)$ (i)________ (ii) ____

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(b) $f_x(P)$ (i)________ (ii) ____

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(c) $f_{xx}(P)$ (i)________ (ii) ____

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(d) $f_{xx}(Q)f_{yy}(Q) - [f_{xy}(Q)]^2$ (i)________ (ii) ______________________________

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7. Throughout this problem we consider the function

$$f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2.$$

You can verify that

$$f_x = 6xy - 12x = 6x(y - 2) \qquad \text{and} \qquad f_{xx} = 6y - 12.$$

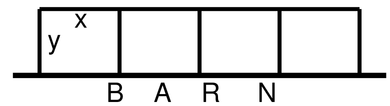
The function f has four critical points.

(a) One critical point is $(0, 0)$. Classify it as a relative maximum, relative minimum, or saddle point.

(b) Find and classify the other three critical points.

8. A farmer will install fences to form four adjacent rectangular pens next to a barn, as shown. Each pen has dimensions x feet by y feet. No fencing is required along the barn.

   In this problem you will use Lagrange multipliers to determine maximum total area the farmer can enclose with a total of 120 feet of fence.

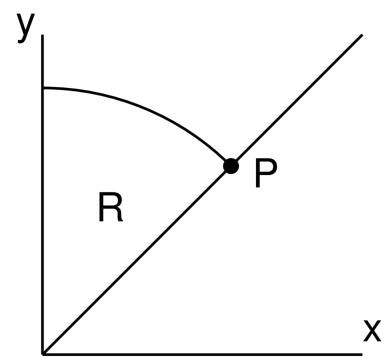


   (a) Express the total area $A(x, y)$ of all the pens in terms of x and y.

   (b) Give the constraint in the form $g(x, y) = k$, where k is a constant.

   (c) Use Lagrange multipliers to find the maximum total area.

9. Let R be the region in the first quadrant that is inside the circle $x^2 + y^2 = 10^2$ of radius 10 and above the line $y = x$.



(a) Give the coordinates of the intersection point P in the indicated coordinate systems.

i. polar: $(r, \theta) =$

ii. rectangular (Cartesian)
$(x, y) =$

(b) Let $f(x, y) = x\sqrt{x^2 + y^2}$ and write

$$\int\int_R f(x, y)\, dA \qquad (*)$$

as an iterated double integral in the indicated coordinate systems.

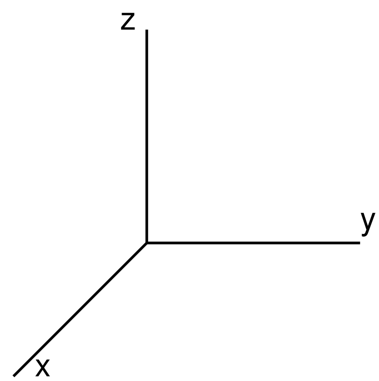
i. rectangular (Cartesian)

ii. polar

(c) Find the exact value of the double integral (*) in part (b).

10. Let E be the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and below the xy-plane.

(a) Sketch the solid E using the given axes.



(b) Let $f(x, y, z) = (x^2 + y^2 + z^2)^3$ and write

$$\int\int\int_E f(x, y, z)\, dV \qquad (*)$$

as an iterated triple integral in the indicated coordinate systems.

i. rectangular (Cartesian)

ii. spherical

(c) Find the exact value of the triple integral (*) in part (b).