Part II Long Answer CALCULATOR ALLOWED

Instructions: A calculator is allowed for Part II of this exam. Write your work directly on this exam. Clearly indicate the places you use a calculator.

1. This problem deals with the point $P = (1, 2, 12)$ and the plane

$$2x + 2y + z = 36.$$

(a) Find a unit vector perpendicular to the plane.

(b) Give parametric equations for the line through $P$ perpendicular to the plane.

(c) Find the point where the line you found in (a) intersects the plane.

(d) Find the distance between the point $P$ and the plane.
2. The plane 

$$2x + 5y + z = 10$$

intersects the $x$-, $y$-, and $z$-axes at the points $A$, $B$, and $C = (0, 0, 10)$, respectively. The figure is not to scale.

(a) Insert the coordinates for points $A$ and $B$ in the figure. 

(b) Compute $\vec{CA} \cdot \vec{CB}$. 

(c) How big is $\angle ACB$ (in degrees or radians)? 

(d) Compute $\vec{CA} \times \vec{CB}$. 

(e) Find the area of $\triangle ABC$. 
3. The velocity of a particle at time $t$ is

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t + 1, 0, 3t^2 \rangle,$$

and its position at time $t = 2$ is $\mathbf{r}(2) = \langle 6, 7, 1 \rangle$. Length is measured in meters, and time is measured in minutes.

(a) Find the speed of the particle at time $t = 2$.

(b) Find parametric equations of the tangent line to the particle's trajectory at $t = 2$.

(c) Find the position $\mathbf{r}(t)$ of the particle at time $t$. 
4. A projectile is fired from a cliff of height 50 m
with initial speed 40 m/sec at an angle
30° (π/6 radians) above horizontal.
The position vector for the projectile is
\[ \mathbf{r}(t) = \langle x(t), y(t) \rangle, \]
as shown. The acceleration due to gravity is
9.8 m/sec^2 directed downward.
Ignore air resistance.
(a) Use the given numbers to find the position vector \( \mathbf{r}(t) \).

(b) Compute the projectile’s time of impact \( t^* \).

(c) i. Compute \( x(t^*) \).

ii. Explain what the number you computed means in words. You can an-
notate and refer to the diagram.

(d) A calculator gives
\[ \int_0^{t^*} |\mathbf{r}'(t)| \, dt = \int_0^{t^*} \sqrt{(x'(t))^2 + (y'(t))^2} \, dt = 228. \]
What does this number mean in the context of this problem?
5. The function \( P = P(r, c) \) gives the monthly profit (in thousands of dollars) earned by a small manufacturer that places \( r \) hours of ads on the radio, and pays a supplier \( c \) dollars for a key component in each product. The table gives some values of \( P \).

\[
\begin{array}{c|ccc}
P(r, c) & c = 50 & c = 60 & c = 70 \\
\hline
r = 1 & 19 & 15 & 11 \\
r = 3 & 23 & 20 & 17 \\
r = 5 & 27 & 25 & 23 \\
\end{array}
\]

(a) Explain in words understandable by a 9th grader what the following assertions mean. Do not use words technical words such as “partial” or “derivative.” Include units.

i. \( P(3, 60) = 20 \)

ii. \( P_c(3, 60) = -0.3 \)

(b) Use the table of values to estimate the partial derivative \( P_r(3, 60) \).

(c) Use a linear approximation (and the values from (a) and (b)) to estimate \( P(4, 62) \).
6. The figure shows the contour diagram for a smooth function \( f(x, y) \) with two points labeled \( P \) and \( Q \). For each expression
(i) state the sign (positive, negative, or zero)
(ii) explain your answer.

(a) \( f(Q) - f(P) \)

(b) \( f_x(P) \)

(c) \( f_{xx}(P) \)

(d) \( f_{xx}(Q)f_{yy}(Q) - [f_{xy}(Q)]^2 \)
7. Throughout this problem we consider the function

\[ f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2. \]

You can verify that

\[ f_x = 6xy - 12x = 6x(y - 2) \quad \text{and} \quad f_{xx} = 6y - 12. \]

The function \( f \) has four critical points.

(a) One critical point is \((0,0)\). Classify it as a relative maximum, relative minimum, or saddle point.

(b) Find and classify the other three critical points.
8. A farmer will install fences to form four adjacent rectangular pens next to a barn, as shown. Each pen has dimensions $x$ feet by $y$ feet. No fencing is required along the barn.

In this problem you will use Lagrange multipliers to determine maximum total area the farmer can enclose with a total of 120 feet of fence.

(a) Express the total area $A(x, y)$ of all the pens in terms of $x$ and $y$.

(b) Give the constraint in the form $g(x, y) = k$, where $k$ is a constant.

(c) Use Lagrange multipliers to find the maximum total area.
9. Let $R$ be the region in the first quadrant that is inside the circle $x^2 + y^2 = 10^2$ of radius 10 and above the line $y = x$.

(a) Give the coordinates of the intersection point $P$ in the indicated coordinate systems.
   
   i. polar: $(r, \theta) =$
   
   ii. rectangular (Cartesian)
   
   $(x, y) =$

(b) Let $f(x, y) = x\sqrt{x^2 + y^2}$ and write

\[
\int \int_R f(x, y) \, dA
\]

as an iterated double integral in the indicated coordinate systems.

i. rectangular (Cartesian)

ii. polar

(c) Find the exact value of the double integral $(*)$ in part (b).
10. Let $E$ be the solid region inside the sphere $x^2 + y^2 + z^2 = 4$ and below the $xy$-plane.

(a) Sketch the solid $E$ using the given axes.

(b) Let $f(x, y, z) = (x^2 + y^2 + z^2)^3$ and write

$$\int \int \int_E f(x, y, z) \, dV$$

as an iterated triple integral in the indicated coordinate systems.

i. rectangular (Cartesian)

ii. spherical

(c) Find the exact value of the triple integral (*) in part (b).