

1. Using a Venn diagram:

	A	A'	
R	0.5	0.2	0.7
R'	0.1	0.2	0.3
	0.6	0.4	1

- (a)  $P(R') = 0.3$
- (b)  $P(R' \cap A') = 0.2$
- (c)  $P(R' \cap A) = 0.1$
- (d)  $P(A|R') = P(A \cap R')/P(R') = 0.1/0.3 = 1/3 = 0.333$
- (e)  $P(K) = P(E)P(K|E) + P(E')P(K|E') = (0.8)(0.75) + (0.2)(0.35) = 0.6 + 0.07 = 0.67$
- (f)  $P(E|K) = P(E \cap K)/P(K) = (0.8)(0.75)/0.67 = 60/67 = 0.8955$

2.  $X$  equals the number of service calls in  $T$  minutes; Poisson with rate  $\lambda = 2.3$  calls per minute. Using TI Stats/List Editor F5 E:

- (a)  $P(X = 0) = 0.1003$
- (b)  $P(X \leq 4) = 0.9162$
- (c)  $\mu = \lambda T = (2.3)(5) = 11.5$  calls in 5 minutes
- (d)  $P(X \leq 15) = 0.8783$
- (e)  $\mu = \lambda T = (2.3)(0.5) = 1.15 : P(X \geq 1) = 1 - P(X = 0) = 1 - 0.3166 = 0.6834$

3.

- (a)  $P(X \leq 2) = f(0) + f(1) + f(2) = 0.9728$
- (b)  $P(X > 1) = f(2) + f(3) + f(4) = 0.1808$
- (c)  $F(1) = P(X \leq 1) = 1 - 0.1808 = 0.8192$  using the preceding part.
- (d)  $\mu = \sum_x x f(x) = (0)(0.4096) + (1)(0.4096) + (2)(0.1536) + (3)(0.0256) + (4)(0.0016) = 0.8$
- (e)  $E(X^2) = \sum_x x^2 f(x) = 1.28$  so  $\sigma^2 = 1.28 - 0.8^2 = 0.64$
- (f)  $\sigma = \sqrt{0.64} = 0.8$

4.  $X$  has a hypergeometric distribution.

(a) 
$$P(X = 0) = \frac{\binom{5}{0} \binom{15}{4}}{\binom{20}{4}} = 0.2817$$

(b) 
$$P(X = 3) = \frac{\binom{5}{3} \binom{15}{1}}{\binom{20}{4}} = 0.0310$$

(c)  $P(X \geq 1) = 1 - P(X = 0) = 0.7183$  using the first part

(d)  $E(X) = n \frac{k}{N} = 4 \frac{5}{20} = 1$  or calculate on TI: 
$$\mu = \sum_x x f(x) = \sum_x x \frac{\binom{5}{x} \binom{15}{4-x}}{\binom{20}{4}}$$

(e) Binomial  $n = 4$  and  $p = 0.25$  on TI Stats/List Editor F5 C:  $P(X \geq 1) \approx 0.6836$  which is not a good approximation because sample size is not much smaller than population size here.

5.

(a)  $P(0.5 \leq X \leq 1) = \int_{0.5}^1 f(x) dx = 219/256 = 0.8555$

(b)  $\mu = \int_{-\infty}^{+\infty} x f(x) dx = \int_0^1 x f(x) dx = 2/3 = 0.6667$

(c)  $E(XY) = \int_0^1 \left( \int_0^1 xy f(x, y) dy \right) dx = 16/45 = 0.3556$

(d)  $P(X \leq Y) = \int_0^1 \left( \int_x^1 f(x, y) dy \right) dx = 23/45 = 0.5111$

(e)  $E(D) = \mu_U - \mu_V = -2.2$

(f)  $\sigma_D^2 = \sigma_U^2 + \sigma_V^2 = 0.9^2 + 0.7^2 = 1.3$  so  $\sigma_D = \sqrt{1.3} = 1.140$

6. Using TI Data/Matrix Editor:

(a)  $P(A_4 \cap D) = P(A_4)P(D | A_4) = (0.1)(0.6) = 0.06$

(b)  $P(A_3 \cap D') = P(A_3)P(D' | A_3) = (0.3)(0.0) = 0$

(c)  $P(D) = \sum_i P(A_i \cap D) = 0.58$

(d)  $P(A_1 | D') = P(A_1 \cap D') / P(D') = 0.7619$

(e)  $P(D_1 \cup D_2) = 1 - P(D'_1 \cap D'_2) = 1 - (0.42)(0.6905) = 0.71$

(f)  $P(A_1 | D'_1 \cap D'_2) = 0.8828$

8	5	8			
9	0	2	3	6	8
10	5	7			
11	0				
12	1	2			
13	2	7			
14					
15	6				

7. (b)  $15.6 - 8.5 = 7.1$  (c)  $\tilde{x} = 10.5$  (d)  $\bar{x} = 10.88$  (e)  $s = 2.082$  (a)

8.

Supplier A:  $X$  is binomial with  $p = 0.95$ ; using TI Stats/List Editor F5 C:

(a)  $P(X \leq 18) = 0.2642$  with  $n = 20$

(b)  $P(X \leq 45) = 0.1036$  with  $n = 50$

Supplier B:  $X$  is normal:

(c)  $P(X \geq 10) = 0.8413$  using F5 #4

(d)  $P(X \leq b) = 0.10$  means  $b = 9.155$  using F5 #2 #1

(e)  $\bar{X}$  is normal with mean 13 and standard deviation  $3/\sqrt{100} = 0.3$  so  $P(12.5 \leq \bar{X} \leq 13.5) = 0.9044$

Supplier C:

(f)  $\hat{x} \pm t_{0.005} \frac{s}{\sqrt{n}} = 12.3 \pm (2.68) \frac{4.2}{\sqrt{50}} = 12.3 \pm 1.59$  or  $10.7 < \mu < 13.9$  is a 99% CI for  $\mu$

Supplier D:

(g)  $\hat{p} \pm z_{0.025} \frac{\sqrt{(\hat{p})(1-\hat{p})}}{\sqrt{n}} = 0.84 \pm (1.96) \frac{\sqrt{(0.84)(0.16)}}{\sqrt{200}} = 0.84 \pm 0.0508$  or  $0.789 < p < 0.891$

is a 99% CI for  $p$