

**SM230 Probability with Naval Applications: Final Examination**

This exam is “closed book and closed notes” except that you may use your own (8 ½ by 11 both sides) resource sheet of paper with your handwritten notes and formulas. You may use your graphing calculator; you are not to borrow a sectionmate’s calculator. You are to give answers as simple fractions or decimal numbers. Please show all work and answers in you blue books or on your solution sheets. Box your answers. Put your name and section number on everything you hand in.

1. The following scenario represents a simple battle analysis.

*First model.* Assume a battle group’s long-range radar will detect an incoming missile with probability  $P(R) = 0.7$  while air patrol will detect it with probability  $P(A) = 0.6$ . The probability is 0.5 that both radar and air patrol will detect the missile.

- (a) Compute the probability that radar will not detect the missile.
- (b) Compute the probability that neither radar nor air patrol will detect the missile.
- (c) Compute the probability that air patrol will detect the missile but radar will not.
- (d) Compute the conditional probability that air patrol detects the missile given radar does not.

*Second model.* Assume that with probability 0.8 a battle group will detect an incoming missile soon enough to attack it with long-range defenses. Long-range defenses have a missile kill probability of 0.75. If early detection does not occur then, the battle group’s probability of a missile kill is 0.35.

- (e) Compute the probability that the battle group will kill the incoming missile.
- (f) The missile is reported killed. Compute the conditional probability that this kill was due to early detection.

2. Service calls come to a maintenance center at the rate of 2.3 calls per minute. Assume these calls arrive according to a Poisson process.

- (a) Compute the probability that there will be no service calls in a one minute interval.
- (b) Compute the probability that there will be 4 or fewer service calls in a one minute interval.
- (c) Compute the mean number of service calls in a 5 minute interval.
- (d) Compute the probability that during a 5 minute interval there will 15 or fewer service calls.
- (e) Compute the probability that when a clock is started, the first service call arrives in less than one-half minute.

3. A brand of SUV tire has been extensively tested over rough terrain and a certain percentage of the tires fail the test. A random sample of 4 of these tires are tested. Let  $X$  be the number of tires that fail the test. Assume that the discrete random variable  $X$  has a distribution given by the following table:

$x$	0	1	2	3	4
$f(x)$	0.4096	0.4096	0.1536	0.0256	0.0016

- (a) Compute  $P(X \leq 2)$ .
- (b) Compute  $P(X > 1)$ .
- (c) Compute the cumulative distribution function value  $F(1)$ .
- (d) Compute the mean  $\mu$  of  $X$ .

(e) Compute the variance  $\left( \sum_x x^2 f(x) \right) - \mu^2$  of  $X$ .

(f) Compute the standard deviation  $\sigma$  of  $X$ .

4. A dealer has 20 truck tires in stock, 5 of which are defective. A mechanic randomly selects 4 of these tires and places them on a truck. Let  $X$  be the number of defectives that are put on the truck.

(a) Compute the probability that no defective tires are placed on the truck.

(b) Compute the probability that 3 defectives tires are placed on the truck.

(c) Compute the probability that at least one defective tire is placed on the truck.

(d) Compute the expected number  $E(X)$  of defective tires that are placed on the truck.

(e) Approximate the answer to part (c) by using a binomial distribution with  $p = 0.25$ .

5. A continuous random variable  $X$  has a *beta* distribution with density function given by  $f(x) = 168x^5(1-x)^2$  for  $0 \leq x \leq 1$  and  $f(x) = 0$  for other values of  $x$ .

(a) Compute  $P(0.5 \leq X \leq 1)$ .

(b) Compute the mean  $\mu$  of  $X$ .

The random variables  $X$  and  $Y$  are jointly continuous with joint probability function

$$f(x, y) = \frac{4}{3}(x^3 + y) \text{ for } 0 \leq x \leq 1, 0 \leq y \leq 1 \text{ and } f(x, y) = 0 \text{ elsewhere.}$$

(c) Compute  $E(XY)$ .

(d) Compute  $P(X \leq Y)$ .

You are given that the random variable  $U$  has mean  $\mu_U = 3.1$  and standard deviation  $\sigma_U = 0.9$  and random variable  $V$  has mean  $\mu_V = 5.3$  and standard deviation  $\sigma_V = 0.7$ . Define  $D = U - V$  and assume that  $U$  and  $V$  are independent random variables.

(e) Compute the mean of  $D$ .

(f) Compute the standard deviation of  $D$ .

6. The location and detection probabilities for the first cycle of a Simple Search are given by the following table.

i	D   A <sub>i</sub>	A <sub>i</sub>
1	0.2	0.4
2	0.7	0.2
3	1.0	0.3
4	0.6	0.1

(a) Compute the probability that the object is located in region 4 and will be detected in the 1<sup>st</sup> search cycle.

(b) Compute the probability that the object is located in region 3 and will not be detected in the 1<sup>st</sup> search cycle.

(c) Compute the probability that the object will be detected during the 1<sup>st</sup> search cycle.

(d) If the object is not detected during the 1<sup>st</sup> cycle, then compute the updated conditional probability  $P(A_1 | D')$  that it is located in region 1.

- (e) Compute the probability that the object will be detected during either the 1<sup>st</sup> or the 2<sup>nd</sup> search cycle.
- (f) If the object is not detected during the 1<sup>st</sup> or 2<sup>nd</sup> search cycle, then compute the conditional probability that it is located in region 1.

7. Fifteen rivets from a certain supplier were tested for breaking strength. The results (in units of 1000 psi) were:

9.3	10.7	8.5	9.6	10.5	15.6	9.2	12.2	9.0	13.2	11.0	8.8	13.7	12.1	9.8
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For this sample:

- (a) carefully draw a stem and leaf plot with stems: 8, 9, 10, 11, 12, 13, 14, 15 ;
- (b) calculate the sample range;
- (c) calculate the sample median  $\tilde{x}$  ;
- (d) calculate the sample mean  $\bar{x}$  ;
- (e) calculate the sample standard deviation  $s$  .

8. A manufacturer looks to purchase rivets that meet the following specification: rivet has breaking strength of at least 10 (in units of 1000 psi).

Supplier A claims that 95% of its rivets will meet the spec. The manufacturer will test this claim by measuring the breaking strength of a sample of size  $n$  randomly selected from a large stock of supplier A produced rivets. Assume supplier A’s claim is true.

- (a) Compute the probability that among a sample of  $n = 20$  supplier A rivets, 18 or fewer will meet the spec.
- (b) Compute the probability that among a sample of  $n = 50$  supplier A rivets, 45 or fewer will meet the spec.

Next, supplier B claims to produce a rivet with random breaking strength  $X$  that has mean  $\mu_x = 13$  and standard deviation  $\sigma_x = 3$ . Assume this claim is true and that  $X$  is normally distributed.

- (c) Compute the probability that one supplier B rivet will meet the spec.
- (d) Ninety percent (90%) of supplier B rivets will have a breaking strength of  $b$  or more. Compute  $b$  .
- (e) Compute the probability  $P(12.5 \leq \bar{X} \leq 13.5)$  that the sample mean  $\bar{X}$  for a random sample of  $n = 100$  supplier B rivets will have a breaking strength within 0.5 of 13.

Next, the manufacturer checks the quality of supplier C rivets by taking a random sample of  $n = 50$  to obtain a sample mean of 12.3 and a sample standard deviation of 4.2.

- (f) Knowing nothing more about supplier C rivets other than these sample statistics, compute a 99% confidence interval for the mean  $\mu$  breaking strength of supplier C rivets.

Lastly, the manufacturer randomly samples  $n = 200$  rivets from supplier D and finds that 168 of them meet spec.

- (g) Compute a 95% confidence interval for the actual proportion  $p$  of supplier D rivets that meet spec.