

Bubble Sheet and Answer Sheet: Write your name, alpha number, and section on your blue book(s), on your answer sheet, and on your Scantron bubble sheet. Bubble in your alpha number in the left-most columns of the Scantron sheet.

Technology: You may use only a TI calculator; you are not to borrow another midshipman's calculator during this exam. No other electronic devices are to be used during the exam.

Resources: The only information you may refer to is that stored on your TI calculator or on the hand-written 8½ by 11 inch (both sides) "information sheet" as specified by your instructor.

When you are done, hand in your information sheet with your answer sheet, bubble sheet, and blue book(s). Make sure that your name, alpha, and section number are on all items.

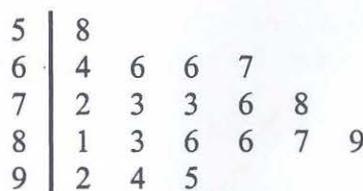
Part One: Multiple Choice (25%). The first 10 problems are multiple-choice.

- i.) Work each problem in your blue book(s),
- ii.) copy the letter of the best or most correct answer on the answer sheet, and
- iii.) fill in the bubble for this letter on the Scantron sheet.

There is no credit and no penalty for a wrong answer.

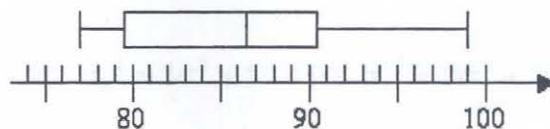
1. The stem and leaf plot to the right displays 19 data values ranging from 58 through 95. The median of this data set equals

- (a) 73
- (b) 76
- (c) 77
- (d) 78
- (e) 89



2. For the data graphically summarized by the box and whisker plot to the right, which of the following numbers equals the third quartile?

- (a) 77.0 (b) 79.5 (c) 86.5
- (d) 90.5 (e) 99.0



3. You are given for events A and B that $P(A) = 0.60$, $P(B) = 0.30$, and $P(A \cap B) = 0.15$.

If you know event A occurs, then what is the conditional probability that event B will occur?

- (a) 0.50 (b) 0.25 (c) 0.18 (d) 0.09 (e) 0.045

4. Exactly one of the following probability distributions is continuous. Which one is it?

- (a) hypergeometric (b) binomial (c) discrete uniform (d) Poisson (e) exponential

5. For a binomial probability distribution with parameters n and p , the product of these two parameters equals the

- (a) variance (b) range (c) median (d) mean (e) standard deviation

6. The lifetime, measured in units of one hundred hours, of a certain battery has the density function

$$f(x) = \frac{1}{2} e^{-x/2} \text{ for } x \geq 0 \text{ (and } f(x) = 0 \text{ for } x < 0).$$

What is the average life of the battery?

- (a) 1/2 hour (b) 1 hour (c) 2 hours (d) 50 hours (e) 200 hours

7. Assume that the number of potholes along any stretch of a certain highway follows a Poisson distribution with a rate of λ potholes per mile. A randomly selected one-mile stretch of the highway is inspected and there are no potholes. With this information, which statement is true about a second randomly selected stretch of this highway.

- (a) The probability of no potholes in the second one-mile stretch of highway equals $\lambda e^{-\lambda}$.
- (b) The probability of no potholes in the second one-mile stretch of highway equals $e^{-\lambda}$.
- (c) The probability of no potholes in the second one-mile stretch of highway equals λ .
- (d) The probability of no potholes in the second one-mile stretch of highway equals $1/\lambda$.
- (e) None of the above.

8. If X is a continuous normal random variable with a mean of 15 and a standard deviation of 3, then $P(X = 18)$ equals

- (a) 0.0000
- (b) 0.0807
- (c) 0.1587
- (d) 0.3414
- (e) 0.8413

9. Assume that two random variables X and Y have joint distribution $f(x, y)$, that X has marginal distribution $g(x)$, and that Y has marginal distribution $h(y)$. If the two random variables are independent, then which statement is *not* correct in general?

- (a) $f(x, y) = g(x)h(y)$
- (b) $E(XY) = E(X)E(Y)$
- (c) $\sigma_{XY} = 0$
- (d) $\sigma_{X+Y} = \sigma_X + \sigma_Y$
- (e) The coefficient of linear correlation $\rho_{XY} = 0$.

10. Assume the random variable X has mean μ and standard deviation σ . What is the variance σ_Y^2 of the random variable $Y = 3X + 5$?

- (a) $\sigma_Y^2 = \sigma^2$
- (b) $\sigma_Y^2 = 3\sigma^2$
- (c) $\sigma_Y^2 = 9\sigma^2$
- (d) $\sigma_Y^2 = 3\sigma^2 + 5$
- (e) $\sigma_Y^2 = 9\sigma^2 + 25$

END OF PART ONE

Part Two: Long Answer (75%). For the following 15 problems,

- i.) show all of your work and answers in your blue book(s) and
- ii.) copy your answers to the answer sheet.

Be sure to reduce numerical answers to simple integers, fractions, or decimals. If a calculator is used, then give the inputs that you entered.

1. The following table lists 10 sample values.

75	70	65	71	73	64	78	81	84	71
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Compute for this data:

- (a) the sample mean;
- (b) the sample median;
- (c) the sample range;
- (d) the sample standard deviation.

2. You are given for events A and B that $P(A) = 0.60$, $P(B) = 0.30$, and $P(A \cap B) = 0.15$.
- What is the probability that either event A or B occurs?
 - What is the probability that both A and B occur?
 - What is the probability that neither A nor B occurs?
 - What is the probability that only A occurs and not B ?
 - What is the probability that one and only one of the events A or B occurs?
 - Are events A and B independent? Explain.
3. A republican governor in a state with no state income tax will appoint a committee to look into possible changes in the income tax law. There are 15 state representatives that could make up the committee. Six of the 15 are democrats and the remaining 9 are republicans. The committee is to consist of 5 members.
- How many different committees are possible?
 - If the 5 members on the committee were selected randomly, what is the probability that the committee consists of 2 democrats and 3 republicans?
 - If the 5 members on the committee were selected randomly, what is the probability that no democrats are on the committee?
 - If the 5 members on the committee were selected randomly, what is the probability that at least one democrat is on the committee?
4. Suppose that 2% of nuclear powered surface ships have defective radars and 5% of conventional powered surface ships have defective radars. Suppose that 40% of all surface ships are nuclear powered and 60% are conventionally powered.
- What is the conditional probability that a ship has a non-defective radar given that it is nuclear powered?
 - What is the probability that a randomly selected ship is nuclear powered and has a defective radar?
 - What is the probability that a randomly selected surface ship has a defective radar?
 - If a randomly selected ship is known to have a defective radar, then what is the probability that this ship is nuclear powered?
5. You are looking for a downed pilot. There is a 40% chance that he is in the forest, a 10% chance that he is in the desert, and a 50% chance that he is in the mountains. If he is in the forest, then the probability you will find him is 0.2. If he is in the desert, then the probability you will find him is 0.9. If he is in the mountains, then the probability you will find him is 0.3.
- You will start by searching all three areas once each — the “1st search cycle”. What is the probability that he is in the mountains and you will find him during the 1st search cycle?
 - What is the probability that you will not find him by the end of the 1st search cycle?
 - If you fail to find the pilot during the 1st search cycle, then what is the probability he is in the mountains?
 - If you search all three areas once each again (2nd search cycle) and do not locate the pilot, then what is the probability he is in the mountains?
6. The discrete random variable X has a probability mass function $f(x)$ given by the following table:

x	-2	0	2	4
$f(x)$	0.216	0.432	0.288	0.064

- Compute the probability $P(X = 0)$.
- Compute the probability $P(X \leq 0)$.

- (c) Compute the mean μ of X .
(d) Compute the standard deviation σ of X .

7. Suppose that a continuous random variable X has a probability density function

$$f(x) = \begin{cases} 3x^2 & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Compute $P(X \leq 0.7)$.
(b) Compute the value of the constant k so that $P(X \leq k) = 0.25$.
(c) Compute the mean $E(X)$.
(d) Compute $E(X^3)$.

8. In a very large inventory of missiles, 25% missiles are known to be defective and will *not* operate as specified. Eight missiles are selected at random from the inventory and tested. Compute the probability that:

- (a) all 8 missiles operate as specified;
(b) none of the 8 missiles operate as specified;
(c) four of the 8 missiles operate as specified;
(d) at least 4 of the 8 missiles operate as specified.

9. The potholes on a major highway occur at the rate of 3.4 per mile. Assume the number of potholes along any stretch of the highway follows a Poisson distribution.

- (a) Compute the probability of no potholes in a one-mile stretch.
(b) Compute the probability of at least 2 potholes in a one-mile stretch.
(c) Compute the expected number of potholes in a five-mile stretch.
(d) Compute the probability of 15 or fewer potholes in a five-mile stretch.

10. The number of calls received by a consumer support service is a Poisson random variable with parameter $\lambda = 6$ calls per hour. Hence, we know that the time between successive calls has an exponential distribution with parameter $\beta = 1/6$ hours per call (or 10 minutes per call).

- (a) What is the probability of waiting at least 15 minutes between any two successive calls?
(b) What is the probability of waiting less than 5 minutes between any two successive calls?
(c) A call has just been received. What is the probability of waiting between 5 and 10 minutes for the next call?

11. Assume that the CQPRs for midshipmen are approximately normally distributed with population mean 3.0 and population standard deviation 0.4.

- (a) Compute the probability that one randomly selected midshipman will have a CQPR at or below 2.5.
(b) Compute the 90th percentile for midshipmen CQPRs. (This is the CQPR below which 90% of the midshipmen CQPRs fall.)
(c) If 10 midshipmen are picked at random, then what is the probability that none of the 10 will have CQPRs at or below 2.5?

12. The joint probability mass function $f(x, y)$ for the discrete random variables X and Y is given by the table below right. Compute:

- (a) $P(X = -1, Y = 2)$;
- (b) $P(X = -1)$;
- (c) $P(X + Y = 3)$;
- (d) $P(X = -1 | Y = 2)$.
- (e) $E(XY)$;
- (f) Copy to your solution sheet the table to the right and fill it in with the marginal distribution of Y .

		x		
		-1	1	
$f(x,y)$	y	0	.05	.20
	2	.20	.15	
	4	.30	.10	

y	0	2	4
$h(y)$			

13. A cigarette manufacturer claims that the amount of nicotine (in mg) in one of its cigarettes is normally distributed with a population mean μ equal to 1.00 mg. Assume this claim to be true.

- (a) Assume that the population standard deviation σ is known to be 0.25 mg. What is the probability that a randomly selected sample of 16 cigarettes has an average nicotine content of more than 1.05 mg?
- (b) Assume now that both μ and σ are unknown. A randomly selected sample of 9 cigarettes has a sample average $\bar{x} = 1.10$ mg of nicotine and sample standard deviation $s = 0.19$ mg. Use this data to find the corresponding 95% confidence interval for the population mean μ .

14. Two companies manufacture golf balls. A driving machine is used to test them. A random sample of 12 golf balls from company #1 yields a sample mean of $\bar{x}_1 = 290$ yds, and an independent random sample of 16 golf balls from company #2 results in a sample mean of $\bar{x}_2 = 300$ yds. Assume that the driving yardage is normally distributed for both brands of golf balls and that the population standard deviations are both $\sigma = 15$ yds.

- (a) Find a 95% confidence interval for the difference in the population mean distances $\mu_2 - \mu_1$.
- (b) Based upon your confidence interval, are you 95% confident that company #2 manufactures a "longer" golf ball than company #1? Explain.

15. Suppose that 100 randomly selected Maryland voters are asked how they feel about smoking in public places and that 70 of the 100 say that they favor a smoking ban in public places.

- (a) Give a 95% confidence interval for p , the true proportion of the entire population of Maryland voters who favor a smoking ban in public places.
- (b) How many randomly selected voters should be asked if we want to be 95% confident that our estimate of p is within 0.01?

END OF PART TWO

When you are done, hand in your information sheet with your answer sheet, bubble sheet, and blue book(s). Make sure that your name, alpha, and section number are on all items.