ENGINEERING MATH SM316

FINAL EXAM

9 MAY 2013 @ 0755

You must show all of your work to receive credit! Round all decimal answers to 4 places unless otherwise specified.

DO TEN (10) OF THE ELEVEN GIVEN PROBLEMS

1. (a) Suppose that 2.5% of the general public has bladder cancer. Quick & Dirty Diagnostics has developed a new test for bladder cancer that correctly identifies the condition in 96% of those who actually have it, and incorrectly identifies bladder cancer in 6% of those who don't have it. If a randomly selected person has a positive Q&D test for bladder cancer, what is the probability that he or she actually has it?

(b) In a poker game, 5 cards are drawn without replacement from an ordinary deck of 52 cards so that all hands are equally likely. Find the probability of getting at least one ace in the hand.

(HINT: First find the probability of getting no ace. Note if you get no ace, how many cards are the 5 drawn from? Then use this to find the desired probability.)

2. Let the discrete random variable (abbreviated here as r.v.) X have probability mass function (pmf), p(x) = P(X = x) given by:

x	-5	-1	0	5
p(x)	.21	.1	.3	k

(a) Find k.

(b) Find the cumulative distribution function (CDF) of X, i.e., $F(x) = P(X \le x)$ - this must be given as a piecewise function defined everywhere, i.e., defined for $-\infty < x < \infty$.

(c) Find $P(-1 \le X \le 7)$, and $P(-1 \le X \le 2 \mid X \ge 0)$.

(d) Find E(X) and Var(X).

3. Given the function

$$f(x) = \begin{cases} kx & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$$

which is the pdf of a continuous r.v. X for $-\infty < x < \infty$.

(a) Find k.

(b) Find and sketch the CDF F(x) of X. F(x) must be given as a piecewise function defined everywhere, i.e., defined for $-\infty < x < \infty$.

(c) Use the CDF to find $P(1 < X < 2)$.

(d) Find E(X) and Var(X).

4 (a) Suppose that the time it takes a professor to get to work is normally distributed with mean 37 minutes and standard deviation 7.5 minutes. If the professor leaves home at 0705, what is the probability that he is late for his 0755 class?

(b) Suppose that 1000 boxes are to be loaded onto an aircraft. All that is known is that the weight of each box is a r.v. with mean 5 lbs and standard deviation 2.5 lb. Assume that the weight of a box is independent from each other box. The aircraft will exceed Gross Takeoff Weight (GTW) if the combined weight of the boxes exceeds 5200 lbs. What is the probability that the aircraft exceeds GTW? Here you must state what Theorem allows you to come through and do this problem.

5. Oil is being pumped through a pipeline. The flow rate (in barrels/min.) is measured once per minute for 36 minutes yielding a sample mean of 100 barrels/min. Also suppose it is known that the population variance, σ^2, is 9.

(a) Find a 99% two sided confidence interval for the mean flow rate μ. This must be done **by hand** showing all work.

(b) With 99% confidence what can you say about the size of the error in using this sample mean to approximate the true mean μ.

(c) For how many minutes would the flow rate have to be measured to be 99% confident that the error incurred by using the average to approximate the true mean μ is less or equal to 1/2 a barrel/min.?

6. (a) Suppose we take a random sample of 25 mids and compute their mean QPR to be 2.65. It is known that QPR's of mid.'s are normally distributed. We also compute the sample variance of the QPR's of our 25 mid.'s to be 1.15. We do not know the population mean or the population variance. Find a 99% confidence interval for the population mean of midshipmen QPR's. You must do this by hand and show all work.

(b) Twenty integrated circuits are drawn from a large stock, 4% of which are defective. What is the probability that two or more of the 20 integrated circuits drawn are defective?

7. (a) Consider the following system

$$\alpha x + y = 4$$
$$x + \alpha y = -2$$

where x, y are variables and α is a parameter.

(i) For what values of α does the system have a unique solution?

(ii) For α satisfying the condition in part (a) write down x and y in terms of α.

(b) Use the <u>definition</u> of linear dependence to show the following vectors in $\mathbb{R}^3$ are linearly dependent:

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix}.$$

8 (a) In the following system, find all solutions if there are any. You must say whether there is a unique solution, or if there are infinitely many solutions, or no solution. If there are more than one solution give 2 different specific solutions.

$$\begin{aligned} x + 3y - 2z &= -7 \\ 4x + y + 3z &= 5 \\ 2x - 5y + 7z &= 19 \end{aligned}$$

(b) Find the LU decomposition for the matrix A below. After you find it, check by matrix multiplication that A = LU.

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 2 & -1 & 4 \\ -1 & 2 & 0 \end{pmatrix}.$$

9. Let $B = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{pmatrix}$. Find **by hand**, the following:

(a) the eigenvalues (eval.'s) of matrix B;
(b) a maximal set of linearly independent eigenvectors (eves.'s). Here evec.'s must have integer components.
(c) Is B diagonalizable? Why or why not?
(**NOTE:** Here you may use your calc. to do det's or rref's. But if you just write down the eval's and evec.'s without any work then you get no credit!)

10 (a) Let the matrix A have characteristic polynomial

$$\Delta(\lambda) = (\lambda-1)(\lambda+2)(\lambda-3)(\lambda+4).$$

Tell what the size of A must be.
(b) For the matrix A in (a) tell whether or not it is diagonalizable and why. If it is so that matrices D and M can be constructed such that $D = M^{-1}AM$ where D is a diagonal matrix. Then tell what specific values appear on the main diagonal of D. Also tell what appears in M - here you can't give specific values because you don't know A.

(c) Let $C = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -2 & 5 \\ 0 & 5 & -2 \end{pmatrix}$. Find **manually**: The eigenvalue corresponding to the eigenvector **v** =

$\begin{bmatrix} 1 \\ 4 \\ -4 \end{bmatrix}$ of the matrix C.

11 (a) Let $G = \begin{pmatrix} 2 & -2 \\ 3 & 2 \end{pmatrix}$ and $H = \begin{pmatrix} 2 & 4 \\ -3 & 2 \end{pmatrix}$. Are G and H similar? Why or why not? (Recall similar means there exists a nonsingular matrix P such that $G = P^{-1}HP$.)

(b) Let $B = \begin{pmatrix} 1 & 0 & -2 & 0 & 0 \\ 2 & 0 & 3 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 \\ 5 & 0 & 4 & 0 & 0 \\ -3 & 4 & -1 & 0 & 2 \end{pmatrix}$. What is the rank of B?

(c) Let $\mathbf{v}_i$ be the ith column ($1 \le i \le 5$) of the matrix B in part (b) of this problem. Are $\mathbf{v}_2$ and $\mathbf{v}_5$ linearly independent? Why or why not?

END