ENGINEERING MATH SM316

FINAL EXAM

1 MAY 2014 @ 1330

You must show all of your work to receive credit! Round all decimal answers to 4 places unless otherwise specified.

PROBABILITY & STATISTICS 120 Points: Do 6 out of 7

1. Consider the experiment of tossing a coin 4 times and each time you get either head (H) or tail (T). Let the random variable (abb. r.v. here) X be the number of H's.
(a) If S is the sample space for this experiment, tell how many elements are in S, $|S|$.
(b) Find $P(1 \le X < 4)$ in these two cases:
(i) coin is fair); (ii) $P(H) = .2$.
(c) If $P(H) = p$ where $0 \le p \le 1$, then find $P(2 \le X < 4)$. Here the answer is to be given in terms of p but all binomial coefficients must be evaluated.

2. A commuter encounters 3 traffic lights each day on his way to work. Let X be the r.v. which represents the number of these that are red lights. The probability mass function, $f(x) = P(X=x)$, of X is

x	0	1	2	3
f(x)	.2	.3	.4	k

(a) Find k.
(b) Find and graph the cumulative distribution function (CDF) of X, i.e., $F(x) = P(X \le x)$. This must be written as a piecewise function defined everywhere.
(c) Find the mean and variance of X, i.e., E(X) and Var(X).

3. Suppose we consider a period of one month (30 days) which the commuter of Problem 2 goes to work and the number of red lights, X_i, he encounters on the i^{th} day, $i=1,2,...,30$, is a r.v. with the distribution given in Problem 2 and this is independent from day to day. Put as usual $\bar{X} = \frac{1}{30} \sum_{i=1}^{30} X_i$.

(a) What can you say the distribution of $\bar{X}$, the average number of red lights per day in the 30 days, is and why?
(b) Find the mean and standard deviation of $\bar{X}$.
(c) Find the probability that the average number of lights is more than 2 per day, i.e., $P(\bar{X} > 2)$.

4. John casually strolls to the train station every day. The time X in minutes he waits for the train is a continuous r.v. with probability density function

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ (1/10)e^{-x/10} & \text{if } x \geq 0 \end{cases}$$

(a) Find and graph the cumulative distribution function CDF, $F(x) = P(X \leq x)$ of X. This must be written as a piecewise function defined everywhere.

(b) (i) Find the probability that John must wait more than 3 minutes for the train.

(ii) Find the probability that John waits at least 6 min's. given that he has already waited at least 3 min.'s. (This is a conditional probability.)

(iii) Compare and comment on your results in parts (i) and (ii). Note that in (ii) if he has to wait at least 6 min.'s assuming that he has already waited at least 3 that means he has to wait at least 3 minutes more!

(c) Find the mean and variance of X, i.e., E(X) and Var(X).

5. At a bottling plant all of the 16 oz. bottles of soda are filled by two machines, Machine 1 (M1) and Machine 2 (M2). M1 has an average fill (μ_1) of 16.21 oz with σ_1 = .14 oz; M2 has an average fill (μ_2) of 16.12 oz with σ_2 = .07 oz. Both the fill amounts are normal r.v.'s. M1, however, fills twice as many bottles as M2 does. (So the probability of being filled by M1, P(M1) is twice the probability of being filled by M2, P(M2).)

(a) Find the portion of bottles from this plant that contain less than 15.96 oz. of soda, i.e. P(fill < 15.96).

(b) What percentage of bottles that contain less than 15.96 oz are filled by M1? That is find the conditional probability

P(M1 | fill < 15.96).

(**HINT**: Make a TVD and use the normal distribution with conditional probability to help fill it in. This is a **Bayes Rule** problem.)

6. An electrical engineer wishes to compare the mean lifetimes of 2 types of transistors. A sample of 60 transistors of type A were found to have a mean lifetime of 1827 hr.'s with a standard deviation σ_A = 168. A sample of 180 transistors of type B were tested and found to have a mean lifetime of 1658 hr.'s with σ_B = 225.

(a) What is the distribution of $\overline{X}_A - \overline{X}_B$ and why?

(b) Find $P(\overline{X}_A - \overline{X}_B \leq 100)$. (**NOTE:** Here you first have to find $E(\overline{X}_A - \overline{X}_B)$ and $StDev(\overline{X}_A - \overline{X}_B)$.)

7. (a) Scores on the math SAT are normally distributed. The ETS

(the maker of the SAT's) claims that the variance in the scores is $\sigma^2 = 48$. A random sample of 41 scores had a sample variance $s^2 = 80$. Find $P(S^2 \geq 80)$ assuming that the claim $\sigma^2 = 48$ is true. What do you think of this claim of the ETS based upon the probability that you just computed?

(b) A metallurgist is studying a new welding process. He manufactures 7 welded joints and measures the yield strength of each. The sample mean is 62.88 and the sample standard deviation is 5.4838. Assume that this random sample is from an approximately normal population. Find a 2-sided 98% confidence interval for μ, the mean yield strength of all welded joints under this new process. This must be done BY HAND showing all steps. You can only use your calculator to check.(If you just write down the answer without work even if it is correct, you get very little credit!)

MATRIX THEORY 80 Points: Do 4 out of 5

8. (a) Use the **definition** of linear independence (abb.l.i.)to show that the following vectors in $\mathbb{R}^3$ are linearly independent:

$$\begin{pmatrix}1\\2\\5\end{pmatrix},\quad \begin{pmatrix}2\\5\\1\end{pmatrix},\quad \begin{pmatrix}1\\5\\2\end{pmatrix}.$$

(b) Define what it means for a set of k n-dimensional vectors $\{X_1, X_2, ..., X_k\}$ to be linearly dependent. Here you can **not** just say they are not l.i. Also say what this means in terms of solving linear systems. Here you must say how many equations and how many unknowns in terms of n and k.

9.(a) Find the rank of the matrix A below

$$A = \begin{pmatrix}1 & 2 & 1 & -2\\2 & 4 & 4 & -3\\3 & 6 & 7 & -4\end{pmatrix}.$$

(b) If one considers the rows of the matrix as vectors in $\mathbb{R}^4$, what does your result of (a) say about whether these vectors are linearly independent or linearly dependent?

(c) Finally if one considers the homogeneous linear system $AX = 0$ where A is as in (a), then tell whether or not this system has a nonzero solution and why.

10. (a) Use <u>manual</u> row reduction **(by hand)** on the augmented matrix to solve the following system. You must show each step of the row reduction - you may use your calculator solely as a check to your solution.

$$\begin{aligned} x + \quad y + 3z &= 1 \\ 2x + 3y - \quad z &= 3 \\ 5x + 7y + \quad z &= 7 \end{aligned}$$

(b) Tell whether the system in (a) has one solution, infinitely many solutions, or no solutions. If it has more than 1, then give two (2) different solutions.

(c) Can the system in (a) be solved by Cramer's Rule? You don't have to do it but you must say why it can be or why it can't be solved this way.

11. Given the matrix C = $\begin{pmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{pmatrix}$.

(a) Find all eigenvalues and a maximal set of linearly independent eigenvectors. (Here abb. eval's and evec's.)

(b) Tell whether or not C is diagonalizable.

(c) If C is diagonalizable, find a nonsingular matrix P (a modal matrix) such that $P^{-1}CP = D$ where D is a diagonal matrix. Here you must say what the matrices P and D are.

12. (a) Suppose the matrix A = $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 2 & -4 \\ -1 & 1 & 4 \end{pmatrix}$ has the eval.'s $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 3$. Is A diagonalizable or not? Why?

(b) Let B = $\begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix}$. Tell why B is diagonalizable without finding any eval.'s or evec.'s.

(c) Are the matrices B and C = $\begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$ similar where B is the matrix in (b). (Here B and C are similar means there is a nonsingular matrix P such that $B = P^{-1}CP$.)

END