ENGINEERING MATH SM316

FINAL EXAM

9 MAY 2012 @ 0755

You must show all of your work to receive credit! Round all decimal answers to 4 places unless otherwise specified.

PROBABILITY & SATISTISTICS 125 Points: Do 5 out of 6

1. Given a bent coin where the probability of heads is p and tails is q. This is a Binomial Experiment. All Binomial Coefficients, if there are any, must be evaluated otherwise you will lose credit.
If we toss the coin 10 times:
(a) What is the probability of getting all heads? This must be in terms of p.
(b) What is the probability of getting at least one tail? This also must be in terms of p.
(c) What is the probability that the first head occurs on the 10th toss?
(d) What is the probability that the first and last toss are heads and the remaining 8 are 3 heads and 5 tails in any order?
(f) What is the probability that there are an equal number of heads and tails?
(f) Assume now that p = 1/10. How many heads do we expect to get in 10 tosses? What is the variance?
(g) Now go back to p and q for probabilities and assume that we toss the coin 100 times. Write an exact expression for the probability of getting at most 2 heads.
(h) Re-do (g) with p = 1/10.

2. Let a continuous random variable X have pdf

$$f(x) = \begin{cases} x \text{ if } 0 \le x \le 1 \\ 1 \text{ if } 1 \le x \le 3/2 \\ 0 \text{ elsewhere} \end{cases}$$

(a) Sketch this pdf.
(b) Find and sketch the CDF $F(x) = P(X \le x)$.
(c) Use the CDF to find $P\left(\frac{1}{2} \le X \le \frac{5}{4}\right)$.
(d) Find the expected value of X, $E(X) = \mu$.
(e) Find the variance of X, $Var(X) = \sigma^2$.

3. (a) If X is a normal random variable with mean μ and standard deviation σ, i.e., it is $n(x;\mu,\sigma)$, then write its pdf and sketch this pdf for $\mu > 0$.
(b) For the special case with mean 0 and standard deviation 1, i.e., the Standard Normal, Z with $n(z;0,1)$, verify the identity that $P(-k \le Z \le k) = 2\ P(Z \le k) - 1$ by using your sketch together with the symmetry properties of the normal.
(c) State the Central Limit Theorem.
(d) Show all steps in constructing an 80% confidence interval for the mean μ of a population with $\sigma^2 = 4$ given a random sample of size $n = 25$ with sample mean $\overline{X} = 2.4$. Here you can <u>not</u> just use the interval function on your calculator.

4. Suppose that a waitress's weekly tips are a random variable with mean \$200 and standard deviation of \$20. Also suppose that her weekly tips are independent from each other. Assume a year has 52 weeks and let X be the random variable which gives her total tips for the year.
(a) What can you say about the distribution of X and why?
(b) Find the mean and standard deviation of X.
(c) What is the probability that her tips for the year exceed \$11,000?
(d) If this waitress wants to buy a car, but she does not want to get in over her head and if she can apply all her tip money for the year to buy her car, how much can she count on in tips for the year with a 90% probability, i.e., find k such that $P(X \ge k) = .90$.

5. Suppose a systems engineer is interested in the mean time required to assemble a printed circuit board. He knows that the standard deviation of assembly time is .45 minutes. He takes a sample of $n = 49$ times and finds the sample means to be $\overline{X} = 95.23$ minutes.
(a) Find a 98% 2-sided confidence interval on the mean time for assembly. You can <u>NOT</u> just do this with the interval function on your calculator, you must show all steps and give the upper and lower confidence limits to 4 decimal places.
(b) What can he say an upper bound for the error is 98% of the time if he uses 95.23 minutes to approximate the true mean assembly time?
(c) How large a sample is required if the engineer wants to be 98% confident that the error in estimating the true mean by the sample mean is less than 0.12 minutes?

6. (a) The range of a certain type of missile is known to have a mean of 5000 meters(m) with a variance of 500 m^2. The range of this type of missile follows a normal distribution. A company claims that a new manufacturing process has decreased the

variance. The Navy has tested a random sample of 16 of these missiles and found the sample variance $S^2 = 350\ m^2$. If the Navy wants to be 95% sure that the variance is still $500\ m^2$, find the values of $\chi^2_{0.975}$ and $\chi^2_{0.025}$ for this sample. Tell whether this confirms or refutes the **company's** claim.

(b) A random sample of 25 plebes had a sample mean $\bar{X} = 660.25$ Math SAT. It is known that the Math SAT's have a normal distribution, but their standard deviation is not known. Find a 95% 2-sided confidence interval for the mean Math SAT score if the sample standard deviation from this sample was 6.25. Here you can <u>NOT</u> just use the interval function on your calculator. You must show all steps in the construction of this confidence interval.

MATRIX THEORY **75 Points: Do 3 out of 4**

7. Given the system

$$\begin{aligned} x + y - z &= 0 \\ x + 2y - 2z &= 8 \\ x + z &= 4. \end{aligned}$$

(a) Rewrite this system in matrix form AX=B where A is a 3×3 matrix.
(b) Find the determinant of A, det(A).
(c) Based on the value of det(A) conclude whether or not the system has a unique solution.
(d) If the system has a unique solution, find it and the inverse of A by using row reduction BY HAND on the appropriate augmented matrix.
(e) If the solution does not have a unique solution then find all solutions by reducing the augmented matrix BY HAND to reduced row echelon form and give two specific solutions.

8. (a) Is the set of vectors

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} \right\}$$

linearly independent? Justify your answer.
(b) For which values of k does the system

$$\begin{aligned} 9x + ky &= 1 \\ kx + y &= 0 \end{aligned}$$

have a unique solution? In the case where the solution is unique, use Cramer's rule to write a formula for x and y in terms of k.

(c) Find the LU decomposition for the matrix

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 0 & 4 \\ -1 & 4 & 2 \end{bmatrix}.$$

After you have found L and U, do the matrix multiplication to check that LU actually does = A.

(d) Explain what the LU decomposition is useful for.

9. Let $M = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$.

(a) Explain how you know without finding any eigenvalue, that M is diagonalizable.

(b) Find BY HAND all eigenvalues and eigenvectors for M.

(c) Find a nonsingular matrix P such that $P^{-1}MP = D$ where D is a diagonal matrix. Also write P^{-1} and do the matrix multiplication to check that $P^{-1}MP$ really is diagonal.

4. Let the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -1 & 3 \end{bmatrix} \text{ and } X = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

(a) Is X an eigenvector of A? If not explain why not. If so, find, BY HAND, the corresponding eigenvalue.

(b) Is A diagonalizable? Why or why not?

(c) Find the rank of A.

THE END