DIRECTIONS: Show all of your work. There are 100 possible points, 5 per problem. Good luck!

1. For each of the following, give a complete definition.
   a. The 11 field properties for the Reals.
   b. A decreasing sequence.
   c. \( \lim_{n \to \infty} a_n = a \)
   d. \( y = f(x) \) is continuous at \( x = 4 \).
   e. An interior point of set \( S \) (a subset of the real numbers).
   f. A subsequential limit of the sequence \( (s_n) \).

2. Give a complete statement of each of the following:
   a. The Archimedean Property.
   b. The Bolzano-Weierstrass Theorem.
   c. The density property for the irrationals.
   d. The intermediate value theorem.

3. For each of the following, either give an example or state why none exists.
   a. A non-empty set \( S \) of real numbers with a lower bound but not an inf.
   b. A non-empty set \( S \) of real numbers with a lower bound but not a minimum.
   c. Two convergent sequences whose quotient diverges.
   d. A Cauchy sequence that is not bounded.
   e. An unbounded sequence with a convergent subsequence.
   f. A function that is continuous on \([0, 1]\) but not uniformly continuous.

4. Prove the following giving a reason for each step.
   a. Every convergent sequence is a Cauchy sequence.
   b. \( \lim_{x \to 2} x^2 + 3x = 10 \) using the \( \varepsilon - \delta \) definition of the limit.
   c. \( f(x) = \frac{1}{x} \) is uniformly continuous on \([2, \infty)\)
   d. Let the sequence \( (a_n) \) be defined by \( a_1 = 1 \) and \( a_{n+1} = \sqrt{2+a_n} \) for all \( n \in \mathbb{N} \).
      Prove that the sequence converges and find its limit.