FALL 2007 FINAL EXAMINATION FOR SM331

11 December 2007, 0755-1055

DIRECTIONS: Show all of your work. There are 100 possible points, 5 per problem. Good luck!

1. For each of the following, give a complete definition.

a. The 11 field properties for the Reals.

b. A decreasing sequence.

c. $\lim_{n\to\infty} a_n = a$

d. $y = f(x)$ is continuous at $x = 4$.

e. An interior point of set S (a subset of the real numbers).

f. A subsequential limit of the sequence (s_n).

2. Give a complete statement of each of the following:

a. The Archimedean Property.

b. The Bolzano-Weierstrass Theorem.

c. The density property for the irrationals.

d. The intermediate value theorem.

3. For each of the following, either give an example or state why none exists.

a. A non-empty set S of real numbers with a lower bound but not an inf.

b. A non-empty set S of real numbers with a lower bound but not a minimum.

c. Two convergent sequences whose quotient diverges.

d. A Cauchy sequence that is not bounded.

e. An unbounded sequence with a convergent subsequence.

f. A function that is continuous on [0, 1] but not uniformly continuous.

4. Prove the following giving a reason for each step.

a. Every convergent sequence is a Cauchy sequence.

b. $\lim_{x\to 2} x^2 + 3x = 10$ using the $\varepsilon - \delta$ definition of the limit.

c. $f(x) = \dfrac{1}{x}$ is uniformly continuous on $[2, \infty)$

d. Let the sequence (a_n) be defined by $a_1 = 1$ and $a_{n+1} = \sqrt{2 + a_n}$ for all $n \in N$.

Prove that the sequence converges and find its limit.