Part I Multiple Choice NO CALCULATOR ALLOWED

Name: _____ **Alpha:** _____ **Instructor:**_____

Instructions: No calculator is allowed for Part I of this exam. Fill in the top part of your Scantron sheet, including the bubbles for your alpha code. Also, bubble in your test's version number, if appropriate. There is room for your work on this exam. Fill in your answers on the bubble sheet. When you are done with Part I, hand in your **bubble sheet** and this **exam** to your instructor, who will give you Part II. You cannot return to Part I once you have handed it in.

Partial credit is available on some problems depending on how "good" your wrong answer is. Example: How many non-empty subsets of $\{a, b, c\}$ are there?

(a) 2 (b) 4 (c) 6 (d) 7 (e) 8

The correct answer is (d). You would get half-credit for (e) because you forgot to omit the empty set.

Part I of this final examination uses the following terminology in graph theory; these might differ from the conventions used in our text and by your instructor in class:

- Unless otherwise specified, the word *graph* refers to an undirected graph with no loops or multiple edges. A *multigraph* allows multiple edges, but no loops.
- A *walk* in a multigraph is a sequence of vertices v_0, v_1, ..., v_k, where each vertex v_i is adjacent to v_{i+1} for $i = 0, 1, \ldots, k-1$. Vertices can be visited more than once, and edges can be used more than once. A walk is *closed* provided $v_0 = v_k$, and is open provided $v_0 \neq v_k$.
- An *Euler walk* in a multigraph is a walk that uses each edge exactly once. An Euler walk can be closed or open.
- A *Hamilton path* in a graph is a walk that visits each vertex exactly once; edges can be unvisited. A *Hamilton cycle* in a graph is a walk that visits each vertex exactly once and returns to the starting vertex.

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1. How many binary sequences of length 4 are there that contain both symbols 0 and 1? For instance, 1101 is one such string, but 0000 is not.

   (a) 16
   (b) 14
   (c) 32
   (d) 8
   (e) none of the above

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2. How many numbers in the set $\{1, 2, 3, \ldots, 210\}$ are divisible by neither 3 nor 7?

   (a) 120
   (b) 110
   (c) 100
   (d) 90
   (e) none of the bove

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3. In how many ways can 6 midshipmen split into two 3-person teams to play volleyball against one another?

   (a) 70
   (b) 20
   (c) 35
   (d) 10
   (e) none of the above

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4. What is the coefficient a_3 of x^3 in the expansion $((1+x)^3)^2 = a_0+a_1x+a_2x^2+a_3x^3+\cdots+a_6x^6$?

   (a) 20
   (b) 28
   (c) 9
   (d) 16
   (e) 56

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5. A fair coin is tossed four times. What is the probability of getting tails exactly twice?

   (a) 1/2
   (b) 3/8
   (c) 5/16
   (d) 1/4
   (e) 3/16

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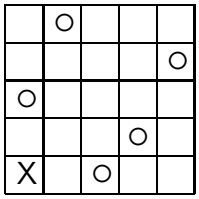
6. In how many ways can we place five markers on a 5-by-5 board with no two markers in the same row or column if the cell in the lower left corner (marked by an X) cannot contain a marker? One such configuration of markers is shown.

   (a) 120
   (b) 24
   (c) 60
   (d) 96
   (e) none of the above



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7. A sequence a_0, a_1, a_2, ... satisfies $a_n = n \cdot a_{n-1} + (-1)^n$ for $n \geq 1$. If $a_5 = 44$, then what is a_3?

   (a) 2
   (b) 9
   (c) 265
   (d) 44
   (e) not determined by the given information

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8. What is the generating function for the sequence

$$a_0 = 4, \quad a_1 = 2, \quad a_2 = 1, \quad a_3 = \frac{1}{2}, \quad a_4 = \frac{1}{4}, \quad \ldots ?$$

(a) $1/(1 - \left(\frac{1}{2}\right) x)$
(b) $4/(1 - 2x)$
(c) $4/(1 + x)$
(d) $4/(1 + 2x)$
(e) $8/(2 - x)$

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9. A palindrome is a string of symbols that reads the same when written in reverse order (like 21412 and ABBA). How many palindromes are in the set $\{100, 101, 102 \ldots, 999\}$?

(a) 72
(b) 90
(c) 100
(d) 81
(e) none of the above

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10. A sequence b_0, b_1, b_2, ... satisfies $b_0 = 2$, $b_1 = 1$ and the recurrence $b_n = 2b_{n-1} - 3b_{n-2}$ for $n \geq 2$. A formula for the sequence has the form $b_n = A_1\alpha_1^n + A_2\alpha_2^n$. Which equation should we solve to find the numbers α_1 and α_2?

(a) $\alpha^2 + 2\alpha - 3 = 0$
(b) $\alpha^2 - 2\alpha + 3 = 0$
(c) $(\alpha - 2)(\alpha - 1) = 0$
(d) $\alpha^2 - 2\alpha - 1 = 0$
(e) none of the above

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11. Which expression is equal to $\binom{20}{15} + \binom{20}{16}$?

(a) $\binom{20}{17}$
(b) $\binom{21}{15}$
(c) $\binom{21}{17}$
(d) $\binom{21}{5}$
(e) none of the above

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12. How many ways permtuations are there for the letters in the multiset {A,L,F,A,L,F,A}? For instance, LAFFLAA is one such permutation.

(a) 210
(b) 420
(c) 630
(d) 105
(e) none of the above

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13. In how many ways can we select a president, a vice-president, and a treasurer from a club with 6 members? A club member cannot hold more than one office.

    (a) 720
    (b) 120
    (c) 20
    (d) 60
    (e) none of the above

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14. We are dealt 13 cards from a standard deck of 52 cards.

    Statement 1: We have four cards of the same suit (e.g., four clubs).
    Statement 2: We have two cards of the same rank (e.g., two jacks).

    (a) Both statement 1 and statement 2 must be true.
    (b) Only statement 1 must be true
    (c) Only statement 2 must be true.
    (d) Neither statement 1 nor statement 2 must be true.
    (e) Both statement 1 and statement 2 must be false.

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15. The ten basketball teams in a league play a round robin tournament. Every team plays every other team once, and no game ends in a tie. Let w_i denote the number of wins for the i-th team ($i = 1, 2, \ldots, 10$). What is the sum $w_1 + w_2 + \cdots + w_{10}$?

    (a) 100
    (b) 90
    (c) 45
    (d) 81
    (e) The sum can vary depending on the results of the games.

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16. Which statement is true?

    (a) The chromatic number of every planar graph is 4.
    (b) Every graph with a Hamilton cycle has a Hamilton path.
    (c) Every graph with a closed Euler walk has an open Euler walk.
    (d) The edge set of the complete bipartite graph $K_{3,3}$ has cardinality 6.
    (e) None of the above statements is true.

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17. The vertex set of a graph is $\{1, 2, \ldots, 9\}$. Vertices i and j are adjacent provided the difference $i - j$ is even. How many edges does the graph have?

    (a) 16
    (b) 20
    (c) 9
    (d) 10
    (e) none of the above

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18. The Registrar wants to create a final exam schedule that uses the smallest number of time-slots with no student assigned two final exams at the same time. Which notion is most closely related to this scenario?

    (a) minimum cost spanning tree of a connected graph
    (b) chromatic number of a graph
    (c) traveling salesperson problem
    (d) a directed graph
    (e) Euler's theorem for planar graphs

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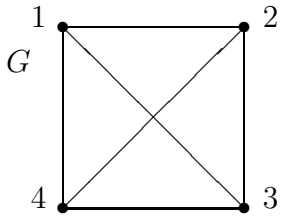
19. The graph G in the diagram has 4 vertices and 6 edges. It does not have a closed Euler walk. However, by adding one or more edges to G we can produce a multigraph that *does* have a closed Euler walk. What is the minimum number of edges we need to add?

    (a) 1
    (b) 2
    (c) 3
    (d) 4
    (e) 6



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20. Which statement is true about the graph G in Problem 19?

    (a) The graph G is both a plane graph and a planar graph.
    (b) The graph G is a plane graph but not a planar graph.
    (c) The graph G is a planar graph but not a plane graph.
    (d) The graph G is neither a plane graph nor a planar graph.
    (e) None of the above statements is true.

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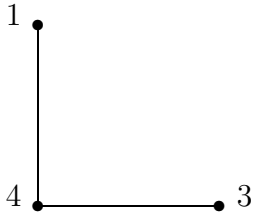
21. How many subgraphs of the graph G in Problem 19 are isomorphic to a path with 3 vertices? One such subgraph is shown here.

    (a) 12
    (b) 24
    (c) 30
    (d) 15
    (e) none of the above



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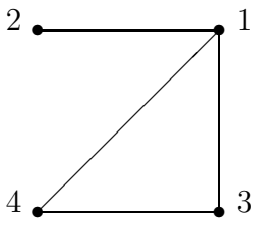
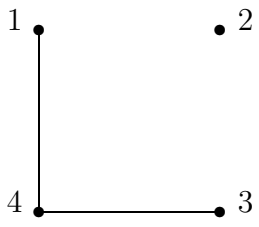
22. How many spanning subgraphs does the graph G in Problem 19 have? Two spanning subgraphs are shown here.

    (a) 32
    (b) 64
    (c) 24
    (d) 1024
    (e) none of the above





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23. In a connected plane graph with 17 regions (including the infinite region) half of the vertices have degree 4, and half have degree 5. How many vertices does the graph have?

    (a) 24

    (b) 12

    (c) 18

    (d) 20

    (e) cannot be determined from the given information

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24. A graph has 6 edges and no cycles of length 3. Its adjacency matrix is

$$\begin{bmatrix} 0 & x & 0 & 0 & 0 & 1 \\ x & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & y \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & y & 1 & 0 & 0 \end{bmatrix}.$$

    What is the ordered pair (x, y)?

    (a) $(x, y) = (1, 0)$

    (b) $(x, y) = (0, 1)$

    (c) $(x, y) = (1, 1)$

    (d) $(x, y) = (0, 0)$

    (e) (x, y) is not determined by the given information.

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25. What is the smallest number of edges in a non-planar graph?

    (a) 6

    (b) 5

    (c) 9

    (d) 10

    (e) none of the above

END OF PART I