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James Oxley

USNA, November 1, 2017

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# A young Bill Tutte

# Selected Papers of W.T. Tutte

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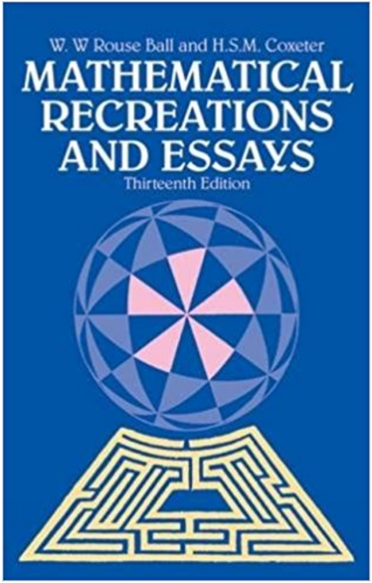
FRS 1987

Officer of the Order of Canada 2001

# Early Interest in Graph Theory



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- Physics: Electrical circuits and Kirchhoff's Laws

"When I became an undergraduate at Trinity College, Cambridge, I already possessed much elementary graph-theoretical knowledge though I do not think I had this knowledge well-organized at the time."

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"As time went on, I yielded more and more to the seductions of Mathematics."

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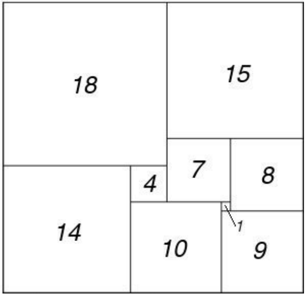
### Problem

*To divide a rectangle into unequal squares.*



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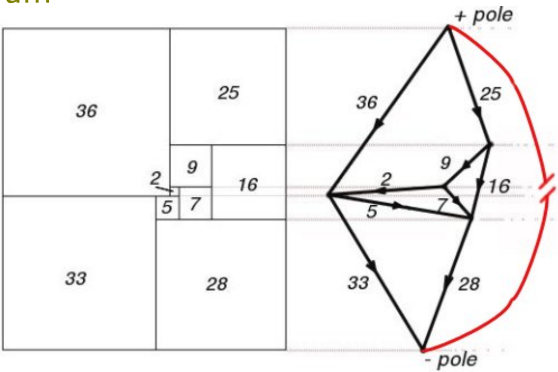
In this paper, "two streams of graph theory from my early studies came together, Kirchhoff's Laws from my Physics lessons, and planar graphs from Rouse Ball's account of the Four Colour Problem."

# Smith diagram



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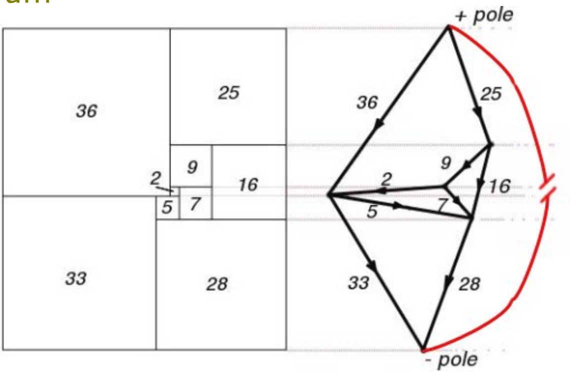


- Horizontal line segments: dots or terminals.

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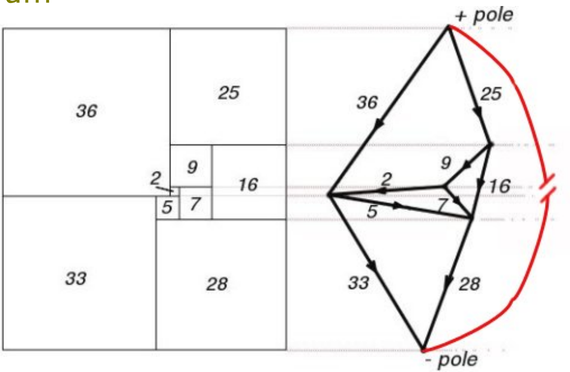


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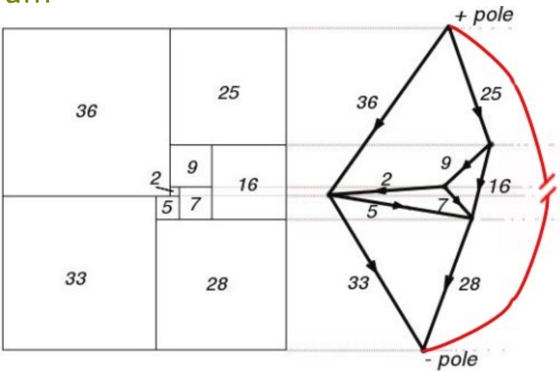


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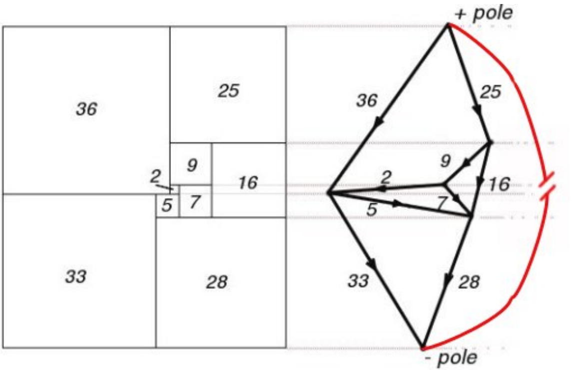


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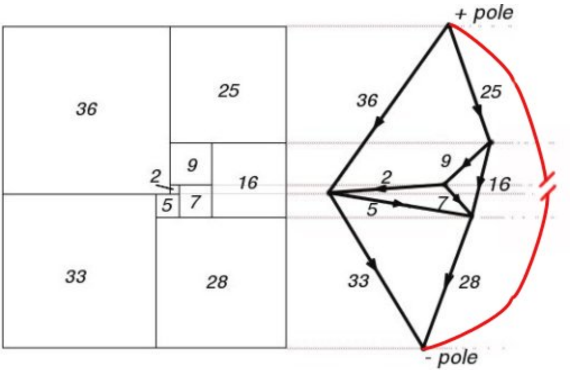


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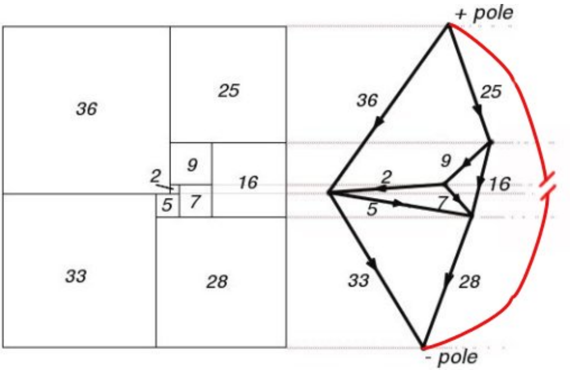


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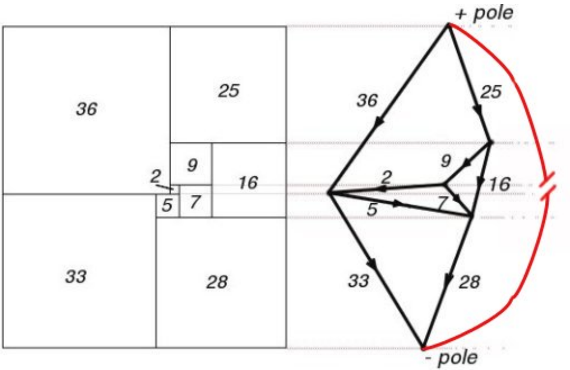


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- All wires have unit resistance. Kirchhoff Laws hold!

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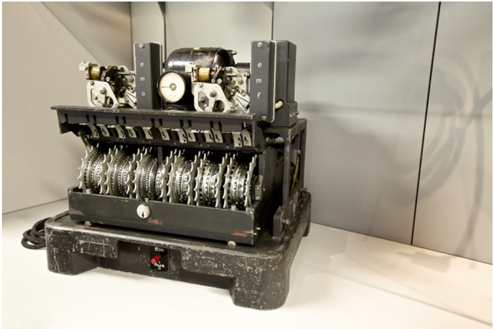

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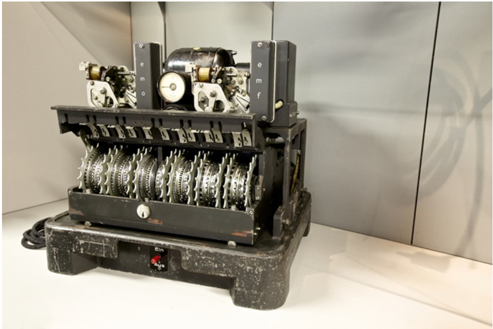

At Bletchley, "I was learning an odd new kind of linear algebra."

# The Bletchley Park Codebreakers

- *Bletchley Park's Lost Heroes*, BBC Two, 2011.

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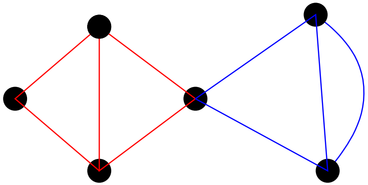
"According to Bletchley Park's historians, General Dwight D. Eisenhower himself described Tutte's work as one of the greatest intellectual feats of the Second World War."

# Connectivity in graphs



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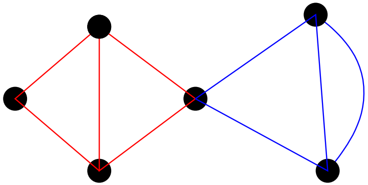

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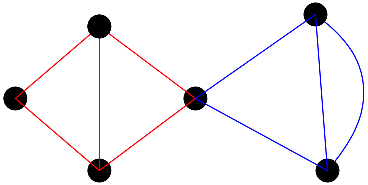

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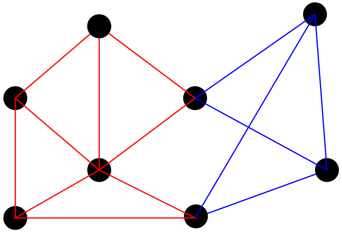

The graph above is 2-connected but not 3-connected.

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**Conjecture (P.G. Tait, 1884)**

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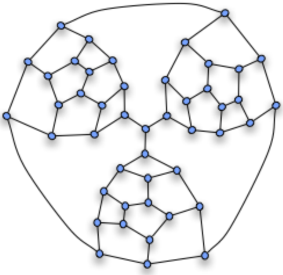
The conjecture fails for the following graph having 46 vertices, 69 edges and 25 faces.



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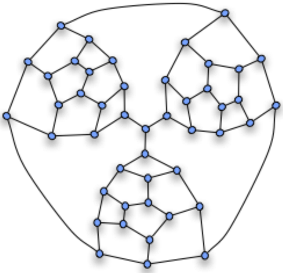

1988: Holton and McKay: Smallest counterexamples: 38 vertices.

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**Theorem (WTT, 1956)**

*Every 4-connected planar graph has a Hamiltonian cycle.*

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Shaun Wylie had advised Tutte to "drop graph theory and take up something respectable, such as differential equations."

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For example, all the vertical edges in a cube.

In a graph $H$, let $q(H)$ be the number of components of $H$ with an odd number of vertices.

**Theorem (W.T. Tutte 1947)**

*A graph $G$ has a 1-factor if and only if, for all $S \subseteq V(G)$,*

$$q(G - S) \leq |S|.$$

# Minors in graphs

A minor of a graph is obtained by a sequence of deletions or contractions of edges or deletions of isolated vertices.

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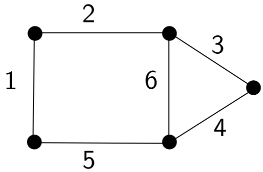
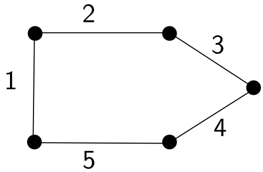
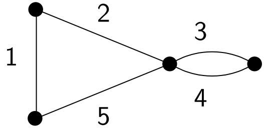



$G$




$G\backslash 6$: the deletion of 6




$G/6$: the contraction of 6

# Tutte's thesis: A ring in graph theory

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1947: *Proc. Cambridge Philosophical Society*

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"I wondered if complexity, or tree number, could be characterized by the above identity alone and decided that it could not."

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## Tutte's thesis: A ring in graph theory

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$P(G\backslash e;k) = P(G;k) + P(G/e;k)$.

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Kirchhoff's Current Law holds at each vertex.

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Younger: "This is matroid theory."

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## Example

Consider the following matrix over the field $GF(2)$.

$$A = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix} \end{array}.$$

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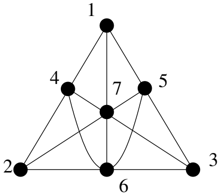

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# Duality

The dual $M^*$ of the matroid $M$ of the $r \times n$ matrix $[I_r|D]$ is the matroid of the matrix $[-D^T|I_{n-r}]$.

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$$U_{2,4} = M[A]$$

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**Theorem (WTT, 1958)**

*A matroid is binary iff it has no $U_{2,4}$-minor.*

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Tutte wrote of this theorem that it "was guided, in the usual vague graph-to-matroid way, by Kuratowski's Theorem and my favourite proof thereof."

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"If a theorem about graphs can be stated in terms of edges and circuits only it probably exemplifies a more general theorem about matroids."

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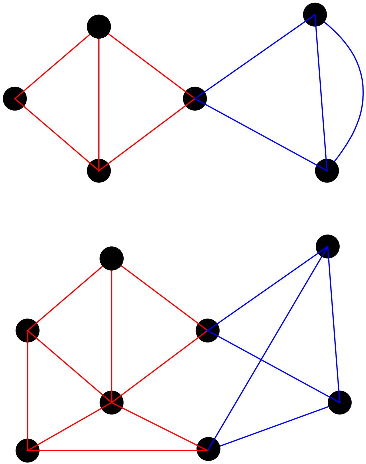

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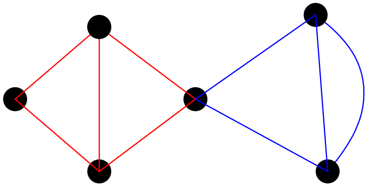

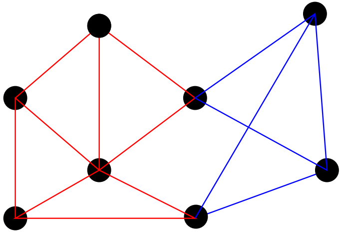

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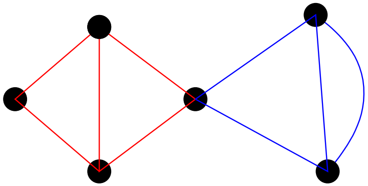

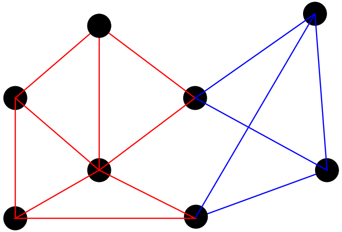

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1980: Seymour's Splitter Theorem generalized this.

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Conjecture (4-Flow Conjecture)

*A graph without cut edges or nowhere-zero 4-flows has a Petersen graph minor.*

1981: Seymour reduced Tutte's Tangential 2-Block Conjecture to the 4-Flow Conjecture by using his Regular Matroids Decomposition Theorem.

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"not only helped protect Britain itself but also shorten the war by an estimated two years, saving countless lives."

# William Thomas Tutte, 1917–2002

# “The three houses problem” by Blanche Descartes

In central Spain in mainly rain
Three houses stood upon the plain.

The houses of our mystery
To which from realms of industry
Came pipes and wires to light and heat
And other pipes with water sweet.

The owners said, “Where these things cross
Burn, leak or short, we’ll suffer loss
So let a graphman living near
Plan each from each to keep them clear.”

Tell them, graphman, come in vain,
They’ll bear the cross that must remain
Explain the planeness of the plain.