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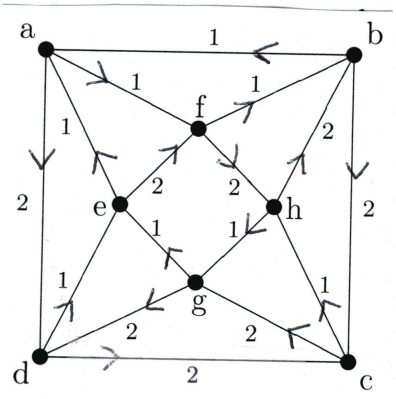
MATH NEWS

Volume XIII, Issue 2, January 2014

USNA *dt*

The Road-Coloring Theorem



You arrive in a strange city to meet a friend. The city has 8 intersections and a confusing network of one-way streets, as shown. You don't have a map. You phone your friend from an intersection, and she gives you these directions to get to her house at h. "See how each is either color 1 or color 2? Just follow the one-way streets according to the color pattern 221 221 221. You'll arrive at my house." "But wait," you say, "I haven't told you where I am now." "Doesn't matter," she replies. "You end up at my house no matter which intersection you start at."

You can verify that if you start at intersection d, say, your path to her house h is $dcgefhgdch$. If you start at b, your path is $bcdefhgdch$. Note that your journey is not the shortest one; in fact, you pass through h at an intermediate stage, but you do indeed arrive at h at the end.

You go to visit your friend again the following weekend. Now she wants to meet you directly at the restaurant at intersection a. She gives you the color-coded instructions 211 211 211. No matter which intersection you start at, you end up at a. **Challenge.** Find the instructions to get to b from any intersection.

This strange phenomenon is not specific to the network for this city. The Road Coloring Theorem asserts that under certain mild hypotheses we can start with any "out k-regular" network of one-way streets and color the roads in so that for any desired destination x there is a set of color-coded instructions that will take you from any starting intersection to x. *Out k-regular* means that k one-way streets start from each intersetion. In the above example $k = 2$. If there are k such streets, then k colors are used for the roads.

The Road Coloring Theorem guarantees *synchronization* and has applications in computer science. For instance, you can hide different parts of a secret file at nodes throughout a network. To re-assemble the file at a desired location at the same time, you simply give the same color-coded instructions to all nodes in the network. In a sophisticated mailroom or manufacturing plant, a robot might be required to orient a box in a specific way before passing it along. Instructions from a road-coloring scheme allow the robot to execute this task without relying on sophisticated sensors to view the starting configuration of each box.

Christopher Cassidy '93: Math Major, Navy SEAL, Astronaut

The critical thinking skills acquired by USNA math majors prepares them for the challenges encountered in their military and professional careers. Christopher Cassidy is an outstanding example. He graduated with the Class of 1993 with a degree in math and later earned a masters degree in ocean engineering from MIT. He served 10 years as a Navy SEAL officer and was awarded two Bronze Stars. In 2004 he started his astronaut career with NASA. He has been to space twice, most recently from March to September 2013, when he conducted a space-walk to make repairs to the International Space Station.

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