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MATH NEWS

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2016: An Abundant Number

Mathematically speaking, the number 2016 is abundant. Midshipmen in the Class of 2016 will not be surprised to learn that the numbers 2013, 2014, and 2015 are deficient. The numbers 2017, 2018, and 2019 are also deficient.

Here are the mathematical details. Let S_n denote the sum of the proper divisors of n . If $S_n > n$, then n is *abundant*; if $S_n < n$, then n is *deficient*. For instance, 10 is deficient because $1 + 2 + 5 = 7 < 10$. Also, 12 is abundant because $1 + 2 + 3 + 4 + 6 = 16 > 12$. It is easy to determine S_n if you know the prime factorization of n . For instance, $12 = 2^2 \cdot 3$ and so

$$S_{12} = (1 + 2 + 2^2)(1 + 3) - 12 = 16,$$

Reason: When you expand the product, each term corresponds to a factor of 12. From the prime factorization $2016 = 2^5 \cdot 3^2 \cdot 7$ we find that the sum of the proper divisors of 2016 is

$$S_{2016} = (1 + 2 + 2^2 + 2^3 + 2^4 + 2^5)(1 + 3 + 3^2)(1 + 7) - 2016 = 4536 > 2016.$$

By the way, if $S_n = n$, then n is *perfect*. The first two perfect numbers are $6 = 1 + 2 + 3$ and $28 = 1 + 2 + 4 + 7 + 14$. Euclid proved that $n = 2^{p-1}(2^p - 1)$ is an even perfect number whenever p and $2^p - 1$ are both prime numbers. The cases $p = 2$ and 3 give $n = 6$ and $n = 28$. Euler showed that the only even perfect numbers are of the form known to Euclid.

Famous hard research problem. Find an odd perfect number. Or show that there aren't any odd perfect numbers.

Here is a more reasonable (and motivational) problem: Everyone knows that $4^{\sqrt{4}} = 16$. But what is

$$4^{\sqrt{4}} + (4 + 4)^{\sqrt{4}} + 44^{\sqrt{4}} ?$$

Mathematics Faculty Profile

LCDR Hetherington is a Surface Warfare Officer from Dallas, Texas. An NROTC graduate of Rensselaer Polytechnic Institute, he holds a B.S. in Applied Physics. He also holds a Masters degree in Engineering Acoustics from the Naval Postgraduate School. His undergraduate work focused on analysis of pulsed laser systems, characterizing dye-pumped lasers and femto-second pulse, frequency-doubled lasers. He is a recipient of the CNO Undersea Warfare Award for his Masters work in nonlinear acoustics characterizing third and fourth order nonlinear effects of acoustic propagation in air. His professional interests include acoustics, wave propagation, nonlinear media, differential equations, applied mathematics, visualization, leadership, educational methods, and public administration.

LCDR Hetherington's afloat assignments include division officer tours on USS HAWES (FFG 53) and USS GEORGE WASHINGTON (CVN 73), and department head tours on USS RUSSELL (DDG 59) and USS VICKSBURG (CG 69). He has deployed on operational tours to the Mediterranean Sea, Arabian Gulf, Western Pacific Ocean, and South China Sea. He is also a qualified Naval Nuclear Engineering Officer.