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MATH NEWS

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Julian Clancy Frazier Memorial Colloquium

Thursday 7 February 1930–2030, Rickover 102

"Putting Topology to Work" by Robert Ghrist, University of Pennsylvania

Latin Squares

A latin square of order n is an n-by-n array of n symbols with each symbol occurring once in each row and once in each column. Here are some examples:

$$\begin{bmatrix} a & b & c & d \\ b & a & d & c \\ c & d & a & b \\ d & c & b & a \end{bmatrix} \qquad \begin{bmatrix} 1 & i & -1 & -i \\ i & -1 & -i & 1 \\ -1 & -i & 1 & i \\ -i & 1 & i & -1 \end{bmatrix} \qquad \begin{bmatrix} 2 & 5 & 3 & 6 & 1 & 4 & 7 \\ 5 & 4 & 2 & 7 & 3 & 1 & 6 \\ 4 & 1 & 7 & 5 & 6 & 2 & 3 \\ 1 & 6 & 4 & 3 & 5 & 7 & 2 \\ 6 & 7 & 5 & 4 & 2 & 3 & 1 \\ 3 & 2 & 6 & 1 & 7 & 5 & 4 \\ 7 & 3 & 1 & 2 & 4 & 6 & 5 \end{bmatrix}$$

Latin squares arise several places in mathematics and applied mathematics:

Abstract algebra. The multiplication table of a *group* is a latin square. For instance, the second latin square of order 4 given above is the multiplication table of the group formed by the four numbers $\{1, i, -1, -i\}$, where $i = \sqrt{-1}$.

Statistics. Statisticians must design experiments carefully to extract the most information from the measured data. For instance, to compare seven different crop fertilizers (1, 2, 3, 4, 5, 6, 7) we should apply the fertilizers in random plots of the field (say, according to the latin square of order 7 given above) in case the soil is naturally better in some parts of the field.

Sudoku. Completed sudoku grids are examples of latin squares of order 9—with additional constraints on the nine 3-by-3 subsquares. The mathematician regards a sudoku puzzle as an instance of a latin square *completion problem*.

Problem. Can the following partial latin squares can be completed to a full latin square of order 5? If so, give a completion; if not, explain why not.

$$\begin{bmatrix} a & b & c & _ & _ \\ b & c & a & _ & _ \\ c & a & b & _ & _ \\ _ & _ & _ & _ & _ \\ _ & _ & _ & _ & _ \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & _ & _ & _ \\ 2 & 3 & 1 & _ & _ \\ _ & 4 & 5 & _ & _ \\ _ & _ & 2 & _ & _ \\ _ & _ & 4 & 3 & 1 \end{bmatrix}$$

Mathematics Faculty Profile

Dr. Nick Loehr is an Associate Professor of Mathematics at the Naval Academy. He is originally from Richmond, Virginia and graduated from Virginia Tech with B.S. degrees (mathematics and computer science) and a Master's degree (computer science). He earned his doctoral degree in mathematics from the University of California at San Diego. His professional interests include algebraic, bijective, and enumerative combinatorics; see his book on these topics in Nimitz Library. He has taught all levels of calculus, as well as proofs, probability, abstract algebra, linear algebra, combinatorics, and cryptography.

Dr. Loehr's previous professional assignments include two-year tours at the University of Pennsylvania (2003–2005) and the College of William and Mary (2005–2007), followed by an ongoing appointment in the Virginia Tech Mathematics Department (2007–present).

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