Mathematics Department

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Profile of the Month
CDR Dave Spoerl

Dave Spoerl was born in Clifton, TN, a small town roughly midway between Memphis and Nashville, but grew up in Wisconsin. From an early age, he demonstrated a "knack" for mathematics, and his elementary school teachers had to give him problems from beyond the curriculum in order to prevent boredom. Spoerl graduated from the U.S. Naval Academy in 1982. Not surprisingly, he was a math major (although he gave systems engineering serious consideration). Among his instructors were Theodore Benac (after whom the Math Lab is named), Fred Davis (now Associate Dean for Academic Affairs), Tom Sanders (now Chairman of the Math Department), Peter Andre, Carol Crawford, and Jim D'Archangelo. His favorite course was probability because of its practical applications, but he also enjoyed computer calculus, because it blended two topics he had previously considered distinct, mathematics and programming. On the other hand, he remembers advanced calculus as "painful". After Flight School in Pensacola, FL, and a tour in Hawaii flying the P-3 Orion, Spoerl completed his M.S. in Operations Analysis at the Naval Postgraduate School in Monterey, CA. At his next stop, Okinawa, Spoerl put the ideas of his Master's thesis into practice working with the Naval Research Laboratory. After a tour at NAS Jacksonville as a flight instructor, Spoerl returned to an operational squadron in Hawaii. During one of his first missions, using the techniques from his thesis, he was able to detect a Russian submarine on his first attempt. This prompted his commanding officer to conclude that the "new guy" was either the luckiest guy in the world, or knew what he was doing! Spoerl admits that luck played a big role. CDR Spoerl returned to the Naval Academy in 1997 as an Instructor of Mathematics. He has taught the calculus sequence, probability and statistics, and all of the operations analysis courses except for SA421 (simulation). His favorite course to teach is SA401 (linear optimization) because, using small-scale examples, he gets to show students methods which apply to large-scale problems of interest to the Navy. Pending approval of the Math Department, CDR Spoerl will in Fall 2007 teach a topics course on integer-programming, which has applications to scheduling, capital budgeting, and manpower problems. Be sure to ask him about it! Spoerl and his wife Margo (an operating room nurse) enjoy traveling and wine-tasting. In 2006 alone, they visited Antigua, Jamaica, Mexico, and the Dominican Republic. While on vacation, they enjoy water sports and "adventure excursions" (climbing, rappelling, etc.). They have two grown children, Doug (22), a Peace Corps volunteer in Africa, and Ashley (20), a junior at Anne Arundel Community College studying graphic design.

Math News

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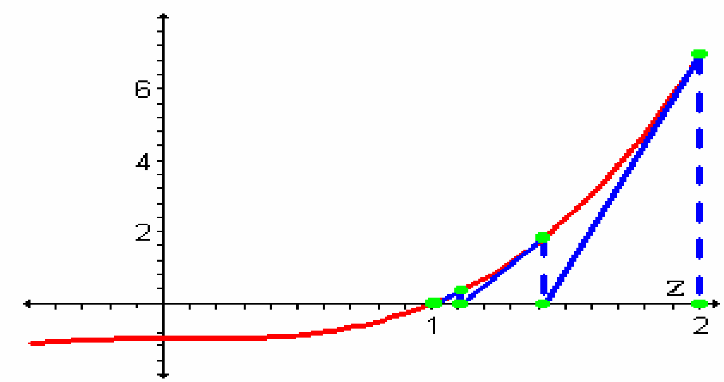
Newton's Method and Complex Numbers

By *Asst Prof. Russell K. Jackson.* In Calculus I, we use Newton's method to find a root (or "zero") of a given function. Although it carries Newton's name, related examples go back *thousands* of years; for instance, the *Babylonian Square Root Method* gives approximations for $\sqrt{\mathbf{A}}$ by starting with an initial guess $\mathbf{z_0}$ and plugging that guess into the formula $\mathbf{z_{n+1}} = \frac{1}{2}(\mathbf{z_n} + \mathbf{A} / \mathbf{z_n})$. Repeatedly cycling each new guess back through this formula produces better and better approximations for $\sqrt{\mathbf{A}}$.

With the advent of computers, Newton's Method has found new life. Recall that the general formula for Newton's Method calls for 1) making an initial guess at a root, $\mathbf{z_0}$, for a given function $\mathbf{f}$, 2) following the tangent line to a better guess, $\mathbf{z_1}$, and 3) repeating Step 2 with each new guess until we're happy that we've gotten close enough. The general formula for the iteration is $\mathbf{z_{n+1}} = \mathbf{z_n} - \mathbf{f}(\mathbf{z_n})/\mathbf{f'}(\mathbf{z_n})$.

For example, for the function $\mathbf{f}(\mathbf{z}) = \mathbf{z}^3 - 1$ and initial guess $\mathbf{z_0} = 2$, these steps



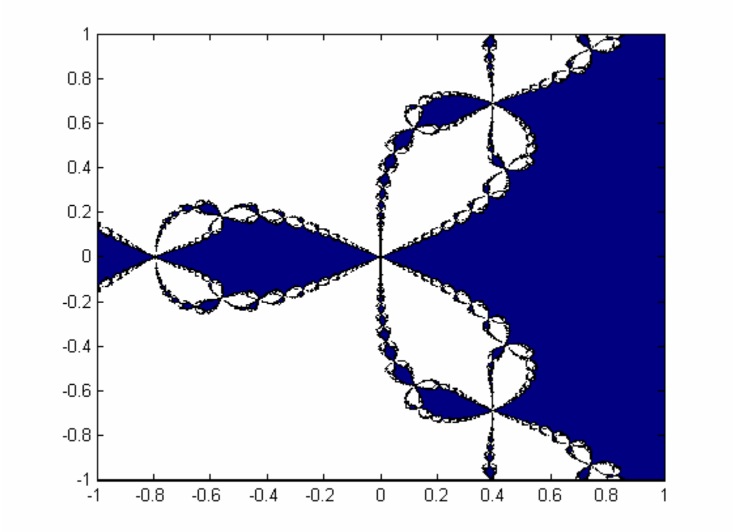
yield $\mathbf{z_0} = 2$, $\mathbf{z_1} \approx 1.4166$, $\mathbf{z_2} \approx 1.1105$, $\mathbf{z_3} \approx 1.0106$…. Continuing, these numbers get closer and closer to the root at 1 (as seen in this picture).

Newton's Method and Complex Numbers (Cont.)

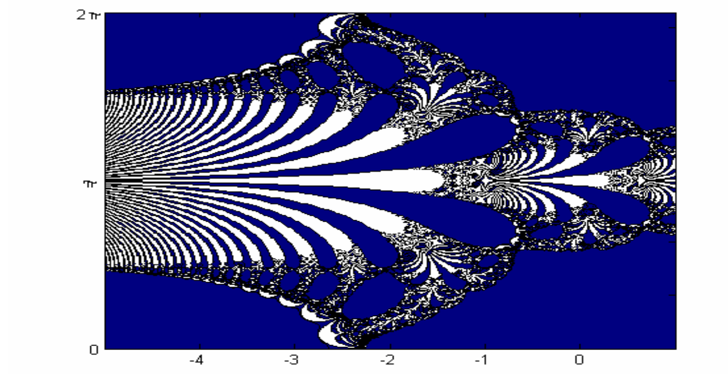
However, if we push our analysis into the *complex* realm (treating **i** = $\sqrt{-1}$ as an actual number), then the function **f**(**z**) = $\mathbf{z}^3 - 1$ has *three* roots, 1 and $(-1 \pm \sqrt{3}\,\mathbf{i})/2$, instead of just the single real root seen in the graph. In fact, in the complex realm, any polynomial of degree **n** has a full cadre of **n** complex roots – this is the *Fundamental Theorem of Algebra*! And, depending upon our initial guess for Newton's Method, we could end up at any one of these roots – or worse yet, end up dividing by zero, or even just bouncing around forever! With the help of computers, it is fun and easy to investigate which initial guesses will approach each different root – and the results of such investigations tend to be beautiful *fractal* objects.

So implementing Newton's method for the function **f**(**z**) = $\mathbf{z}^3 - 1$, where will any initial guess take us? The answer is given pictorially below – in this picture, each point in the plane represents a complex number. The real part is given by the horizontal position and the imaginary part by the vertical. Here, the points that eventually converge to the root **z**=1 are colored dark, and all other points are left white.

For even more fun, we use Newton's method to search for roots of the *ex-*



Newton's Method and Complex Numbers (Cont.)



ponential function **f**(**z**) = $e^z - 1$. This function has a real root at **z** = 0, and purely imaginary roots at **z** = 2**nπ i**, for all integers **n**. Above, points that eventually converge to some root are colored dark, and points that never converge to any root are left white.

FOOD FOR THOUGHT:
"The whole of science is nothing more than the refinement of everyday thinking." **Albert Einstein,** from "Physics and Reality," Franklin Institute Journal 221, no. 3 (March 1936), pp. 349-382.

Answer for last issue question: What is my house number ? Ans. 12634. MIDN 4/C Casey Howsare and MIDN 4/C Anna Bernal are last month's winners. Please stop by the Mathematics Department to claim your prizes.

Question of the Month

Half an hour ago it was twice as long after noon as it is from now until midnight. What time is it now?
E-mail your answer to Prof. Garcia smg@usna.edu. Among those with the right answer, a randomly chosen midshipman will get to choose between a fantastic math water bottle or a cool koozie.

The symbol used on the head of Math News is one of the Platonic Solids. Ref. http://mathworld.wolfram.com/PlatonicSolid.html.